MATHEMATICS

Courses offered by the Department of Mathematics are listed under the subject code MATH on the Stanford Bulletin’s ExploreCourses web site.

The Department of Mathematics offers programs leading to the degrees of Bachelor of Science, Master of Science, and Doctor of Philosophy in Mathematics, and also participates in the program leading to the B.S. in Mathematical and Computational Science, and the M.S. and Ph.D. degree programs offered through the Institute for Computational & Mathematical Engineering.

Mission of the Undergraduate Program in Mathematics

The mission of the undergraduate program in Mathematics is to provide students with a broad understanding of mathematics encompassing logical reasoning, generalization, abstraction, and formal proof. Courses in the program teach students to create, analyze, and interpret mathematical models and to communicate sound arguments based on mathematical reasoning and careful data analysis. The mathematics degree prepares students for careers in the corporate sector, tech industry, government agencies, and for graduate programs in mathematics.

Learning Outcomes (Undergraduate)

The department expects undergraduate majors in the program to be able to demonstrate the following learning outcomes. These learning outcomes are used in evaluating students and the department’s undergraduate program. Students are expected to demonstrate:

1. problem solving skills,
2. the ability to formulate proofs and to structure mathematical arguments,
3. the ability to communicate mathematical ideas via extended written presentation.

Advanced Placement in Mathematics

Students can receive units of advanced placement credit for single-variable calculus, depending on their scores on the CEEB Advanced Placement Examination or the IB Exam. See the the "Advanced Placement (http://exploredegrees.stanford.edu/undergraduatedegreesandprograms/#aptext)” section of this bulletin.

Those who have not studied single-variable calculus or have studied it partially but are not ready to begin with multivariable calculus (MATH 50-series) should begin at the single-variable course recommended by the math placement diagnostic.

Students who are ready to study multivariable calculus (based on prior coursework or exams, or recommended by the math placement diagnostic) should begin with one of the following courses:

<table>
<thead>
<tr>
<th>Course</th>
<th>Title</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH 51</td>
<td>Linear Algebra and Differential Calculus of Several Variables</td>
<td>5</td>
</tr>
<tr>
<td>MATH 52</td>
<td>Integral Calculus of Several Variables</td>
<td>5</td>
</tr>
<tr>
<td>MATH 53</td>
<td>Ordinary Differential Equations with Linear Algebra</td>
<td>5</td>
</tr>
</tbody>
</table>

The above sequence supplies the necessary mathematics background for most majors in science and engineering. It also provides a solid foundation for the major or minor in Mathematics, or in Mathematical and Computational Science.

Bachelor of Science in Mathematical and Computational Science

The Department of Mathematics participates with the departments of Computer Science, Management Science and Engineering, and Statistics in a program leading to a B.S. in Mathematical and Computational Science. See the “Mathematical and Computational Science (http://exploredegrees.stanford.edu/schoolofhumanitiesandsciences/mathematicalandcomputationalscience)” section of this bulletin.

Introductory and Undergraduate Courses

The department offers a year-long sequence in single-variable calculus: MATH 19, MATH 20, and MATH 21. In addition, the department offers a faster-paced version of the same material in two quarters: MATH 41, MATH 42.

There are three options for studying multivariable mathematics:

1. MATH 51, MATH 52, and MATH 53 cover differential and integral calculus in several variables, linear algebra, and ordinary differential equations. These topics are taught in an integrated fashion and emphasize applications. MATH 51 covers differential calculus in several variables and introduces matrix theory and basic linear algebra; MATH 52 covers integral calculus in several variables and vector analysis; MATH 53 studies further topics in linear algebra and applies them to ordinary differential equations. This sequence is strongly recommended for incoming freshmen who have mastered single-variable calculus.
2. The sequence MATH 61CM, MATH 62CM, and MATH 63CM (Modern Mathematics: Continuous Methods) covers the material of the Math 50 series at a much more advanced level with an emphasis on rigorous proofs and conceptual arguments.
3. The sequence MATH 61DM, MATH 62DM, and MATH 63DM (Modern Mathematics: Discrete Methods) covers the same linear algebra material as the Math 60CM series and otherwise focuses on topics in discrete mathematics, algebra, and probability theory at an advanced level with an emphasis on rigorous proofs.

Learning Outcomes (Graduate)

The master’s degree is conferred upon candidates who have developed advanced knowledge and skills in Mathematics. This is achieved through completion of courses, in the primary field as well as related areas, and experience with independent work and specialization.

The Ph.D. is conferred upon candidates who have demonstrated substantial scholarship and the ability to conduct independent research and analysis in Mathematics. Through completion of advanced course work and rigorous skills training, the doctoral program prepares students to make original contributions to the knowledge of Mathematics and to interpret and present the results of such research.

Bachelor of Science in Mathematics

Units

The following degree requirements are in addition to the University's basic requirements for the bachelor's degree.

Students wishing to major in Mathematics must satisfy the following requirements and complete a minimum of 64 units:

1. Department of Mathematics courses totaling at least 49 units credit; such courses must be taken for a letter grade. For the purposes of this requirement, STATS 116 Theory of Probability, PHIL 151 Metalogic, and PHIL 152 Computability and Logic count as Department of Mathematics courses.
2. Additional courses taken from Department of Mathematics courses numbered 101 and above or from approved courses in other
Mathematics

disciplines with significant mathematical content, totaling at least 15 units credit. At least 9 of these units must be taken for a letter grade.

3. The Department of Mathematics adviser can be any member of the department's faculty.

4. To receive the department's recommendation for graduation, a student must have been enrolled as a major in the Department of Mathematics for a minimum of two full quarters, including the quarter immediately before graduation.

Students with an Advanced Placement score of 5 in BC math may receive 10 units credit and fulfill requirement '1' by taking at least 39 units of Department of Mathematics courses. Students with an Advanced Placement score of at least 4 in BC math or 5 in AB math may receive 8 units credit and fulfill requirement '1' by taking at least 41 units of Department of Mathematics courses.

Sophomore seminar courses may be counted among the choice of courses under item '1'. Other variations of the course requirements laid down above (under items '1' and '2') may, in some circumstances, be allowed. For example, students transferring from other universities may be allowed credit for some courses completed before their arrival at Stanford. However, at least 24 units of the 49 units under item '1' above and 9 of the units under item '2' above must be taken at Stanford. In all cases, approval for variations in the degree requirements must be obtained from the department's Director of Undergraduate Studies. The policy of the Mathematics Department is that no courses other than the MATH 60 series and below may be double-counted toward any other University major or minor.

**Proof Writing**

For students who are not experienced with writing mathematical proofs, this crucial skill can be learned by taking any one of the following courses: MATH 104 Applied Matrix Theory, MATH 110 Applied Number Theory and Field Theory, MATH 113 Linear Algebra and Matrix Theory, or MATH 115 Functions of a Real Variable (after finishing the chosen calculus sequence).

**Preparation for Graduate School:**

It is to be emphasized that the above regulations are minimum requirements for the major; students contemplating graduate work in mathematics are strongly encouraged to include the courses MATH 116 Complex Analysis, MATH 120 Groups and Rings, MATH 121 Galois Theory, MATH 147 Differential Topology or MATH 148 Algebraic Topology, and MATH 171 Fundamental Concepts of Analysis in their selection of courses, and in addition, take at least three Department of Mathematics courses over and above the minimum requirements laid out under items '1' and '2' above, including at least one 200-level course. Such students are also encouraged to consider the possibility of taking the honors program.

**Sample Course Plans**

To help develop a sense of the type of course selection (under items '1' and '2') that would be recommended for math majors with various backgrounds and interests, see the following examples. These represent only a few of a very large number of possible combinations of courses that could be taken in fulfillment of the Mathematics major requirements:

**Example 1: for students with both pure and applied interests**

<table>
<thead>
<tr>
<th>Units</th>
<th>Course</th>
<th>Number of Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single-variable calculus: AP Credit, (MATH 19, MATH 20, MATH 21) or 8-10 (MATH 41, MATH 42)</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Multivariable calculus: (MATH 51, MATH 52, MATH 53) or MATH 113 Linear Algebra and Matrix Theory</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MATH 104 Applied Matrix Theory</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MATH 106 Functions of a Complex Variable</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MATH 109 Applied Group Theory</td>
<td>3</td>
</tr>
</tbody>
</table>

**Example 2: for students with strong theoretical interest and considering graduate school in math**

<table>
<thead>
<tr>
<th>Units</th>
<th>Course</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single-variable calculus: AP Credit, (MATH 19, MATH 20, MATH 21) or 8-10 (MATH 41, MATH 42)</td>
</tr>
<tr>
<td></td>
<td>Select one of the following multivariable calculus sequences:</td>
</tr>
<tr>
<td></td>
<td>Math 50 Series: (MATH 51, MATH 52, MATH 53)</td>
</tr>
<tr>
<td></td>
<td>Math 60CM Series: (MATH 61CM, MATH 62CM, MATH 63CM)</td>
</tr>
<tr>
<td></td>
<td>Math 60DM Series: (MATH 61DM, MATH 62DM, MATH 63DM)</td>
</tr>
</tbody>
</table>

**Total Units: 64-66**

The courses from other departments are only meant as examples; there are many suitable courses in several departments that can be taken to fulfill part or all of requirement '2'.

In addition, those contemplating eventual graduate work in Mathematics should consider including at least one graduate-level math course such as MATH 205A Real Analysis, MATH 210A Modern Algebra I, or MATH 215A Algebraic Topology or MATH 215B Differential Topology. Such students should also consider the possibility of entering the honors program.
Example 3: for students interested in applied math
Students desiring significant computational and/or financial and/or statistical components are encouraged to also consider the Mathematics and Computational Science program.

Single-variable calculus: AP Credit, (MATH 19, MATH 20, MATH 21) or 8-10 (MATH 41, MATH 42)

Select one of the following multivariable calculus sequences:
Math 50 Series (MATH 51, MATH 52, MATH 53). These courses are not proof-oriented.
Math 60 Series (MATH 61DM, MATH 62DM, MATH 63DM). This proof-oriented sequence is called Modern Mathematics: Discrete Methods

Honors Program

This option is intended for students who have strong theoretical interests and abilities in mathematics. The goal is to give students a thorough introduction to the main branches of mathematics. The honors program requires a senior thesis, which can involve either original research or expository work on advanced topics in mathematics. This option provides an excellent background with which to enter a Master’s or Ph.D. program in Mathematics. Students completing the honors program are awarded a B.S. in Mathematics with Honors.

It is recommended that either sequence (MATH 61CM, MATH 62CM, MATH 63CM) or (MATH 61DM, MATH 62DM, MATH 63DM) be taken in the freshman year. To graduate with a B.S. in Mathematics with Honors, the following conditions apply in addition to the usual requirements for math majors:

1. The selection of courses under items ‘1’ and ‘2’ above must contain:

   MATH 106 Functions of a Complex Variable 3
   or MATH 116 Complex Analysis 3
   MATH 120 Groups and Rings 3
   MATH 171 Fundamental Concepts of Analysis 3

   And must also include seven additional 3-unit Math courses numbered 121 or higher. (The logic courses PHIL 151 Metalogic and PHIL 152 Computability and Logic can also be used.) These seven courses must include at least:

   One Algebra Course:
   MATH 121 Galois Theory 3
   MATH 122 Modules and Group Representations 3
   MATH 152 Elementary Theory of Numbers 3
   MATH 154 Algebraic Number Theory 3

2. All courses counting towards the honors requirements (MATH 106/MATH 116, MATH 120, MATH 171, all 7 additional Math courses used to fulfill the major requirement, and MATH 197) must be taken for a letter grade.

3. Students must have an average GPA of at least 3.0 across all math classes counting towards the major at the time of applying for honors to be eligible for acceptance into the honors program, as well as upon graduation to graduate with honors.

4. Majors interested in honors can apply in winter quarter of their junior year at the earliest, and no later than the last day of classes in the spring quarter of junior year.

5. Students in the honors program must write a senior thesis. In order to facilitate this, the student must, by the end of the junior year, choose an undergraduate thesis adviser from the Department of Mathematics faculty and map out a concentrated reading program under the direction and guidance of the adviser. This will occur when the student applies for honors. During the senior year, the student must enroll in MATH 197 Senior Honors Thesis with his/her adviser for a total of 6 units (recommended to be spread over three quarters), and work toward completion of the thesis under the direction and guidance of the thesis adviser. The thesis may contain original material, or be a synthesis of work in current or recent research literature. The 6 units of credit for MATH 197 Senior Honors Thesis are required in addition to the 64 units required of the major. (See the major requirements at the top of the page.)

6. The deadline for the senior thesis final draft is the Monday of week 8 of the student’s graduation quarter.

In addition to the minimum requirements laid out above, it is strongly recommended that students take at least one graduate-level course (that is, at least one course in the 200 plus range). MATH 205A Real Analysis, MATH 210A Modern Algebra I, and MATH 215A Algebraic Topology or MATH 215B Differential Topology are especially recommended in this text.

Students with questions about the honors program should see the department’s director of undergraduate studies.

Minor in Mathematics

To qualify for the minor in Mathematics, a student should complete, for a letter grade, at least six Department of Mathematics courses numbered 51 or higher, totaling a minimum of 24 units. For the purposes of this requirement, STATS 116 Theory of Probability, PHIL 151 Metalogic, and PHIL 152 Computability and Logic count as Department of Mathematics courses. No other courses from outside the Department of Mathematics may be used towards the minor in Mathematics.

It is recommended that these courses include:
Math Minor

<table>
<thead>
<tr>
<th>Course</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH 51</td>
<td>5</td>
</tr>
<tr>
<td>or MATH 61CM</td>
<td></td>
</tr>
<tr>
<td>or MATH 61DM</td>
<td></td>
</tr>
<tr>
<td>MATH 52</td>
<td>5</td>
</tr>
<tr>
<td>or MATH 62CM</td>
<td></td>
</tr>
<tr>
<td>or MATH 62DM</td>
<td></td>
</tr>
<tr>
<td>MATH 53</td>
<td>5</td>
</tr>
<tr>
<td>or MATH 63CM</td>
<td></td>
</tr>
<tr>
<td>or MATH 63DM</td>
<td></td>
</tr>
<tr>
<td>Plus three additional MATH courses</td>
<td>9</td>
</tr>
<tr>
<td><strong>Total Units</strong></td>
<td><strong>24</strong></td>
</tr>
</tbody>
</table>

At least 12 of the units applied toward the minor in Mathematics must be taken at Stanford. The policy of the Mathematics Department is that no courses other than the MATH 50/60 series and below may be double-counted toward any other University major or minor.

Master of Science in Mathematics

The University's basic requirements for the master's degree are discussed in the "Graduate Degrees (http://exploredegrees.stanford.edu/graduatedegrees)" section of this bulletin. Students should pay particular attention to the University's course requirements for graduate degrees.

University Coterminal Requirements

Coterminal master's degree candidates are expected to complete all master's degree requirements as described in this bulletin. University requirements for the coterminal master's degree are described in the "Coterminal Master's Program (http://exploredegrees.stanford.edu/cotermdegrees)" section. University requirements for the master's degree are described in the "Graduate Degrees (http://exploredegrees.stanford.edu/graduatedegrees/#masterstext)" section of this bulletin.

After accepting admission to this coterminal master's degree program, students may request transfer of courses from the undergraduate to the graduate career to satisfy requirements for the master's degree. Transfer of courses to the graduate career requires review and approval of both the undergraduate and graduate programs on a case by case basis.

In this master's program, courses taken three quarters prior to the first graduate quarter, or later, are eligible for consideration for transfer to the graduate career. No courses taken prior to the first quarter of the sophomore year may be used to meet master's degree requirements.

Course transfers are not possible after the bachelor's degree has been conferred.

The University requires that the graduate adviser be assigned in the student's first graduate quarter even though the undergraduate career may still be open. The University also requires that the Master's Degree Program Proposal be completed by the student and approved by the department by the end of the student's first graduate quarter.

The following are specific departmental requirements:

Candidates must complete an approved course program of 45 units of courses beyond the department requirements for the B.S. degree, of which at least 36 units must be Mathematics Department courses, taken for a letter grade. The Mathematics Department courses must include at least 18 units numbered 200 or above. The candidate must have a grade point average (GPA) of 3.0 (B) or better in courses used to satisfy the Master's degree requirement. In addition, the student must pass qualifying examinations given by the department.

To be admitted to candidacy, the student must have successfully completed 27 units of graduate courses (that is, courses numbered 200 and above). In addition, the student must pass qualifying examinations given by the department.

Beyond the requirements for candidacy, the student must complete a course of study approved by the Graduate Affairs Committee of the Department of Mathematics and submit an acceptable dissertation. In accordance with University requirements, Ph.D. students must complete a total of 135 course units beyond the bachelor's degree. These courses should be Department of Mathematics courses or approved courses from other departments. The course program should display substantial breadth in mathematics outside the student's field of application. The student must receive a grade point average (GPA) of 3.0 (B) or better in courses used to satisfy the Ph.D. requirement. In addition, the student must pass the Department area examination and the University oral examination.

Experience in teaching is emphasized in the Ph.D. program. Each student is required to complete nine quarters of such experience. The nature of the teaching assignment for each of those quarters is determined by the department in consultation with the student. Typical assignments include teaching or assisting in teaching an undergraduate course or lecturing in an advanced seminar.

For further information concerning degree programs, fellowships, and assistantships, inquire of the department's student services office.

Ph.D. Minor in Mathematics

Requirements for the Ph.D. Minor in Mathematics are:

Complete both of the following Sequences: 1, 2

**Sequence 1**

<table>
<thead>
<tr>
<th>Course</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH 106</td>
<td>3</td>
</tr>
<tr>
<td>or MATH 116</td>
<td></td>
</tr>
<tr>
<td>MATH 131P</td>
<td>3</td>
</tr>
<tr>
<td>MATH 132</td>
<td></td>
</tr>
</tbody>
</table>

**Sequence 2**

<table>
<thead>
<tr>
<th>Course</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH 113</td>
<td>3</td>
</tr>
<tr>
<td>MATH 120</td>
<td>3</td>
</tr>
<tr>
<td>or MATH 152</td>
<td>3</td>
</tr>
</tbody>
</table>

**Additional Courses**

<table>
<thead>
<tr>
<th>Courses</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 units of 200-level MATH</td>
<td>21</td>
</tr>
<tr>
<td>Total Units</td>
<td>33</td>
</tr>
</tbody>
</table>

The Financial Mathematics M.S. degree program is no longer offered through the School of Humanities and Sciences. The Institute for Computational and Mathematical Engineering (ICME (https://icme.stanford.edu)) now offers a master's degree track in Mathematical and Computational Finance (http://exploredegrees.stanford.edu/schoolofengineering/instituteforcomputationalandmathematicalengineering/#masterstext).

Doctor of Philosophy in Mathematics

The University's basic requirements for the doctorate (residence, dissertation, examinations, etc.) are discussed in the "Graduate Degrees (http://exploredegrees.stanford.edu/graduatedegrees)" section of this bulletin. The following are specific departmental requirements.

To be admitted to candidacy, the student must have successfully completed 27 units of graduate courses (that is, courses numbered 200 and above). In addition, the student must pass qualifying examinations given by the department.

Beyond the requirements for candidacy, the student must complete a course of study approved by the Graduate Affairs Committee of the Department of Mathematics and submit an acceptable dissertation. In accordance with University requirements, Ph.D. students must complete a total of 135 course units beyond the bachelor's degree. These courses should be Department of Mathematics courses or approved courses from other departments. The course program should display substantial breadth in mathematics outside the student's field of application. The student must receive a grade point average (GPA) of 3.0 (B) or better in courses used to satisfy the Ph.D. requirement. In addition, the student must pass the Department area examination and the University oral examination.

Experience in teaching is emphasized in the Ph.D. program. Each student is required to complete nine quarters of such experience. The nature of the teaching assignment for each of those quarters is determined by the department in consultation with the student. Typical assignments include teaching or assisting in teaching an undergraduate course or lecturing in an advanced seminar.

For further information concerning degree programs, fellowships, and assistantships, inquire of the department's student services office.

1. MATH 106 Functions of a Complex Variable
2. MATH 116 Complex Analysis
3. MATH 131P Partial Differential Equations
4. MATH 113 Linear Algebra and Matrix Theory
5. MATH 120 Groups and Rings
6. MATH 152 Elementary Theory of Numbers
7. 21 units of 200-level MATH courses
The 100-level courses may have been completed during undergraduate study, and their equivalents from other universities are acceptable.

A third coherent sequence designed by the student, subject to the approval of the graduate committee, may be considered as a substitute for Sequence 1 or 2.

The 200-level courses must be taken at Stanford and approved by the Department of Mathematics Ph.D. minor adviser.

Emeriti: Gregory Brumfiel, Gunnar Carlsson, Robert Finn, Yitzhak Katznelson, Georg Kreisel, Harold Levine, Tai-Ping Liu, R. James Milgram, Donald Ornstein, Richard Schoen, Leon Simon

Chair: Eleny Ionel

Professors: Daniel Bump, Emmanuel Candès, Sourav Chatterjee, Ralph L. Cohen (on leave), Brian Conrad, Amir Dembo (on leave Autumn), Persi Diaconis, Yakov Eliashberg, Jacob Fox, Soren Galatius (on leave Autumn, Winter), Eleny Ionel, Steven Kerckhoff, Jun Li, Rafe Mazzeo, Maryam Mirzakhani, George Papanicolaou, Lenya Ryzhik, Kannan Soundararajan, Ravi Vakil, Andras Vasy, Akshay Venkatesh, Brian White (on leave), Lexing Ying

Associate Professor: Jan Vondrak

Assistant Professors: Thomas Church (on leave), Jonathan Luk

Acting Professor: Alex Wright

Szego Assistant Professors: Riddhipratim Basu, Alexei Entin, Laura Fredrickson, Yu Gu, Or Herskavit, Michael Kemeny, Fredrick R W M Manners, Jenny Wilson, Tian Yang, Xiuwen Zhu

Lecturers: Jorge Acosta, Susie Kimport, Mark Lucianovic, George Schaeffer, Wojciech Wieczorek

Adjunct Professors: Brian Conrey, David Hoffman

Courses

MATH 19. Calculus. 3 Units.
Introduction to differential calculus of functions of one variable. Review of elementary functions (including exponentials and logarithms), limits, rates of change, the derivative and its properties, applications of the derivative. Prerequisites: trigonometry, advanced algebra, and analysis of elementary functions (including exponentials and logarithms). You must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

MATH 20. Calculus. 3 Units.
The definite integral, Riemann sums, antiderivatives, the Fundamental Theorem of Calculus, and the Mean Value Theorem for integrals. Integration by substitution and by parts. Area between curves, and volume by slices, washers, and shells. Initial-value problems, exponential and logistic models, direction fields, and parametric curves. Prerequisite: Math 19 or equivalent. If you have not previously taken a calculus course at Stanford then you must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

MATH 21. Calculus. 4 Units.
Review of limit rules, sequences, functions, limits at infinity, and comparison of growth of functions. Review of integration rules, integrating rational functions, and improper integrals. Infinite series, special examples, convergence and divergence tests (limit comparison and alternating series tests). Power series and interval of convergence, Taylor polynomials, Taylor series and applications. Prerequisite: Math 20 or equivalent. If you have not previously taken a calculus course at Stanford then you must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

MATH 41. Calculus. 5 Units.
Introduction to differential and integral calculus of functions of one variable. Topics: limits, rates of change, the derivative and applications, introduction to the definite integral and integration. Math 41 and 42 cover the same material as Math 19-20-21, but in two quarters rather than three. Prerequisites: trigonometry, advanced algebra, and analysis of elementary functions, including exponentials and logarithms. *If you have not previously taken a calculus course at Stanford then you must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

Same as: Accelerated

MATH 41A. Calculus ACE. 6 Units.
Students attend MATH 41 lectures with different recitation sessions, four hours instead of two, emphasizing engineering applications. Prerequisite: application; see http://soe.stanford.edu/edp/programs/ace.html.

MATH 42. Calculus. 5 Units.
Continuation of 41. Methods of symbolic and numerical integration, applications of the definite integral, introduction to differential equations, infinite series. Prerequisite: 41 or equivalent. *If you have not previously taken a calculus course at Stanford then you must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

Same as: Accelerated

MATH 42A. Calculus ACE. 6 Units.
Students attend MATH 42 lectures with different recitation sessions, four hours instead of two, emphasizing engineering applications. Prerequisite: application; see http://soe.stanford.edu/edp/programs/ace.html.

MATH 51. Linear Algebra and Differential Calculus of Several Variables. 5 Units.
Geometry and algebra of vectors, matrices and linear transformations, eigenvalues of symmetric matrices, vector-valued functions and functions of several variables, partial derivatives and gradients, derivative as a matrix, chain rule in several variables, critical points and Hessian, least-squares, constrained and unconstrained optimization in several variables, Lagrange multipliers. Prerequisite: 21, 42, or the math placement diagnostic (offered through the Math Department website) in order to register for this course.

MATH 51A. Linear Algebra and Differential Calculus of Several Variables, ACE. 6 Units.
Students attend MATH 51 lectures with different recitation sessions: four hours per week instead of two, emphasizing engineering applications. Prerequisite: application; see http://soe.stanford.edu/edp/programs/ace.html.

MATH 52. Integral Calculus of Several Variables. 5 Units.
Iterated integrals, line and surface integrals, vector analysis with applications to vector potentials and conservative vector fields, physical interpretations. Divergence theorem and the theorems of Green, Gauss, and Stokes. Prerequisite: 51 or equivalents.

MATH 53. Ordinary Differential Equations with Linear Algebra. 5 Units.
Ordinary differential equations and initial value problems, systems of linear differential equations with constant coefficients, applications of second-order equations to oscillations, matrix exponentials, Laplace transforms, stability of non-linear systems and phase plane analysis, numerical methods. Prerequisite: 51 or equivalents.
MATH 61CM. Modern Mathematics: Continuous Methods. 5 Units.
This is the first part of a theoretical (i.e., proof-based) sequence in multivariable calculus and linear algebra, providing a unified treatment of these topics. Covers general vector spaces, linear maps and duality, eigenvalues, inner product spaces, spectral theorem, metric spaces, differentiation in Euclidean space, submanifolds of Euclidean space, inverse and implicit function theorems, and many examples. Part of the linear algebra content is covered jointly with Math 61DM. Students should know 1-variable calculus and have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on the BC-level Advanced Placement calculus exam, or consent of the instructor.

MATH 61DM. Modern Mathematics: Discrete Methods. 5 Units.
This is the first part of a theoretical (i.e., proof-based) sequence in discrete mathematics and linear algebra. Covers general vector spaces, linear maps and duality, eigenvalues, inner product spaces, spectral theorem, counting techniques, and linear algebra methods in discrete mathematics including spectral graph theory and dimension arguments. Part of the linear algebra content is covered jointly with Math 61CM. Students should have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on the BC-level Advanced Placement calculus exam, or consent of the instructor.

This sequence is not appropriate for students planning to major in natural sciences, economics, or engineering, but is suitable for majors in any other field (such as MCS (“data science”), computer science, and mathematics).

MATH 62CM. Modern Mathematics: Continuous Methods. 5 Units.
A continuation of themes from Math 61CM, centered around: manifolds, multivariable integration, and the general Stokes' theorem. This includes a treatment of multilinear algebra, further study of submanifolds of Euclidean space and an introduction to general manifolds (with many examples), differential forms and their geometric interpretations, integration of differential forms, Stokes' theorem, and some applications to topology. Prerequisite: Math 61CM.

MATH 62DM. Modern Mathematics: Discrete Methods. 5 Units.
This is the second part of a proof-based sequence in discrete mathematics. This course covers topics in elementary number theory, group theory, and discrete Fourier analysis. For example, we'll discuss the basic examples of abelian groups arising from congruences in elementary number theory, as well as the non-abelian symmetric group of permutations. Prerequisites: 61DM or 61CM.

MATH 63CM. Modern Mathematics: Continuous Methods. 5 Units.
A proof-based course on ordinary differential equations, continuing themes from Math 61CM and Math 62CM. Topics include linear systems of differential equations and necessary tools from linear algebra, stability and asymptotic properties of solutions to linear systems, existence and uniqueness theorems for nonlinear differential equations with some applications to manifolds, behavior of solutions near an equilibrium point, and Sturm-Liouville theory. Prerequisites: Math 61CM and Math 62CM.

MATH 63DM. Modern Mathematics: Discrete Methods. 5 Units.
Third part of a proof-based sequence in discrete mathematics. This course covers several topics in probability (random variables, independence and correlation, concentration bounds, the central limit theorem) and topology (metric spaces, point-set topology, continuous maps, compactness, Brouwer's fixed point and the Borsuk-Ulam theorem), with some applications in combinatorics. Prerequisites: 61DM or 61CM.

MATH 70SI. The Game of Go: Strategy, Theory, and History. 1 Unit.
Strategy and mathematical theories of the game of Go, with guest appearance by a professional Go player.

MATH 80Q. Capillary Surfaces: Explored and Unexplored Territory. 3 Units.
Preference to sophomores. Capillary surfaces: the interfaces between fluids that are adjacent to each other and do not mix. Recently discovered phenomena, predicted mathematically and subsequently confirmed by experiments, some done in space shuttles. Interested students may participate in ongoing investigations with affinity between mathematics and physics.

MATH 87Q. Mathematics of Knots, Braids, Links, and Tangles. 3 Units.
Preference to sophomores. Types of knots and how knots can be distinguished from one another by means of numerical or polynomial invariants. The geometry and algebra of braids, including their relationships to knots. Topology of surfaces. Brief summary of applications to biology, chemistry, and physics.

MATH 101. Math Discovery Lab. 3 Units.
MDL is a discovery-based project course in mathematics. Students work independently in small groups to explore open-ended mathematical problems and discover original mathematics. Students formulate conjectures and hypotheses; test predictions by computation, simulation, or pure thought; and present their results to classmates. No lecture component; in-class meetings reserved for student presentations, attendance mandatory. Admission is by application: http://math101.stanford.edu. Motivated students with any level of mathematical background are encouraged to apply. WIM.

MATH 104. Applied Matrix Theory. 3 Units.
Linear algebra for applications in science and engineering: orthogonality, projections, spectral theory for symmetric matrices, the singular value decomposition, the QR decomposition, least-squares, the condition number of a matrix, algorithms for solving linear systems. (Math 113 offers a more theoretical treatment of linear algebra.) Prerequisites: Math 51 and programming experience on par with CS106BnMath 104 and EE103/CME103 cover complementary topics in applied linear algebra. The focus of Math 104 is on algorithms and concepts; the focus of EE103 is on a few linear algebra concepts, and many applications.

MATH 106. Functions of a Complex Variable. 3 Units.
Complex numbers, analytic functions, Cauchy-Riemann equations, complex integration, Cauchy integral formula, residues, elementary conformal mappings. (Math 116 offers a more theoretical treatment.) Prerequisite: 52.

MATH 107. Graph Theory. 3 Units.
An introductory course in graph theory establishing fundamental concepts and results in variety of topics. Topics include: basic notions, connectivity, cycles, matchings, planar graphs, graph coloring, matrix-tree theorem, conditions for hamiltonicity, Kuratowski’s theorem, Ramsey and Turan-type theorem. Prerequisites: 51 or equivalent and some familiarity with proofs is required.

MATH 108. Introduction to Combinatorics and Its Applications. 3 Units.
Topics: graphs, trees (Cayley’s Theorem, application to phylogeny), eigenvalues, basic enumeration (permutations, Stirling and Bell numbers), recurrences, generating functions, basic asymptotics. Prerequisites: 51 or equivalent.

MATH 109. Applied Group Theory. 3 Units.
Applications of the theory of groups. Topics: elements of group theory, groups of symmetries, matrix groups, group actions, and applications to combinatorics and computing. Applications: rotational symmetry groups, the study of the Platonic solids, crystallographic groups and their applications in chemistry and physics. Honors math majors and students who intend to do graduate work in mathematics should take 120. WIM.

MATH 110. Applied Number Theory and Field Theory. 3 Units.
Number theory and its applications to modern cryptography. Topics: congruences, finite fields, primality testing and factorization, public key cryptography, error correcting codes, and elliptic curves, emphasizing algorithms. WIM.
MATH 113. Linear Algebra and Matrix Theory. 3 Units.
Algebraic properties of matrices and their interpretation in geometric terms. The relationship between the algebraic and geometric points of view and matters fundamental to the study and solution of linear equations. Topics: linear equations, vector spaces, linear dependence, bases and coordinate systems; linear transformations and matrices; similarity; eigenvectors and eigenvalues; diagonalization. (Math 104 offers a more application-oriented treatment.)

MATH 114. Introduction to Scientific Computing. 3 Units.
Introduction to Scientific Computing Numerical computation for mathematical, computational, physical sciences and engineering: error analysis, floating-point arithmetic, nonlinear equations, numerical solution of systems of algebraic equations, banded matrices, least squares, unconstrained optimization, polynomial interpolation, numerical differentiation and integration, numerical solution of ordinary differential equations, truncation error, numerical stability for time dependent problems and stiffness. Implementation of numerical methods in MATLAB programming assignments. Prerequisites: MATH 51, 52, 53; prior programming experience (MATLAB or other language at level of CS 106A or higher). Graduate students should take it for 3 units and undergraduate students should take it for 4 units. Same as: CME 108

MATH 115. Functions of a Real Variable. 3 Units.
The development of real analysis in Euclidean space: sequences and series, limits, basic functions, derivatives, integrals. Basic point set topology. Honors math majors and students who intend to do graduate work in mathematics should take 171. Prerequisite: 51.

MATH 116. Complex Analysis. 3 Units.
Analytic functions, Cauchy integral formula, power series and Laurent series, calculus of residues and applications, conformal mapping, analytic continuation, introduction to Riemann surfaces, Fourier series and integrals. (Math 106 offers a less theoretical treatment.) Prerequisites: 52, and 115 or 171.

MATH 118. Mathematics of Computation. 3 Units.
Notions of analysis and algorithms central to modern scientific computing: continuous and discrete Fourier expansions, the fast Fourier transform, orthogonal polynomials, interpolation, quadrature, numerical differentiation, analysis and discretization of initial-value and boundary-value ODE, finite and spectral elements. Prerequisites: MATH 51 and 53.

MATH 120. Groups and Rings. 3 Units.
Recommended for Mathematics majors and required of honors Mathematics majors. Similar to 109 but altered content and more theoretical orientation. Groups acting on sets, examples of finite groups, Sylow theorems, solvable and simple groups. Fields, rings, and ideals; polynomial rings over a field; PID and non-PID. Unique factorization domains. WIM.

MATH 121. Galois Theory. 3 Units.
Field of fractions, splitting fields, separability, finite fields. Galois groups, Galois correspondence, examples and applications. Prerequisite: Math 120 and (also recommended) 113.

MATH 122. Modules and Group Representations. 3 Units.
Modules over PID. Tensor products over fields. Group representations and group rings. Maschke's theorem and character theory. Character tables, construction of representations. Prerequisite: Math 120. Also recommended: 113.

MATH 131P. Partial Differential Equations. 3 Units.
An introduction to PDE; particularly suitable for non-Math majors. Topics include physical examples of PDE's, method of characteristics, D'Alembert's formula, maximum principles, heat kernel, Duhamel's principle, separation of variables, Fourier series, Harmonic functions, Bessel functions, spherical harmonics. Students who have taken MATH 171 should consider taking MATH 173 rather than 131P. Prerequisite: 53.

MATH 136. Stochastic Processes. 3 Units.

MATH 137. Mathematical Methods of Classical Mechanics. 3 Units.

MATH 138. Celestial Mechanics. 3 Units.
Mathematically rigorous introduction to the classical N-body problem: the motion of N particles evolving according to Newton's law. Topics include: the Kepler problem and its symmetries; other central force problems; conservation theorems; variational methods; Hamilton-Jacobi theory; the role of equilibrium points and stability; and symplectic methods. Prerequisites: 53, and 115 or 171.

MATH 142. Hyperbolic Geometry. 3 Units.
An introductory course in hyperbolic geometry. Topics may include: different models of hyperbolic geometry, hyperbolic area and geodesics, isometries and Mobius transformations, conformal maps, Fuchsian groups, Farey tessellation, hyperbolic structures on surfaces and three manifolds, limit sets. Prerequisites: some familiarity with the basic concepts of differential geometry and the topology of surfaces and manifolds is strongly recommended.

MATH 143. Differential Geometry. 3 Units.
Geometry of curves and surfaces in three-space and higher dimensional manifolds. Parallel transport, curvature, and geodesics. Surfaces with constant curvature. Minimal surfaces.

MATH 145. Algebraic Geometry. 3 Units.
Hilbert's nullstellensatz, complex affine and projective curves, Bezout's theorem, the degree/genus formula, blow-up, Riemann-Roch theorem. Prerequisites: 120, and 121 or knowledge of fraction fields. Recommended: familiarity with surfaces equivalent to 143, 146, 147, or 148.

MATH 146. Analysis on Manifolds. 3 Units.
Differentiable manifolds, tangent space, submanifolds, implicit function theorem, differential forms, vector and tensor fields. Frobenius' theorem, DeRham theory. Prerequisite: 62CM or 52 and familiarity with linear algebra and analysis arguments at the level of 113 and 115 respectively.

MATH 147. Differential Topology. 3 Units.
Smooth manifolds, transversality, Sards' theorem, embeddings, degree of a map, Borsuk-Ulam theorem, Hopf degree theorem, Jordan curve theorem. Prerequisite: 115 or 171.

MATH 148. Algebraic Topology. 3 Units.
Fundamental group, covering spaces, Euler characteristic, homology, classification of surfaces, knots. Prerequisite: 109 or 120.

MATH 151. Introduction to Probability Theory. 3 Units.
Counting; axioms of probability; conditioning and independence; expectation and variance; discrete and continuous random variables and distributions; joint distributions and dependence; central limit theorem and laws of large numbers. Prerequisite: 52 or consent of instructor.

MATH 152. Elementary Theory of Numbers. 3 Units.
Euclid's algorithm, fundamental theorems on divisibility; prime numbers; congruence of numbers; theorems of Fermat, Euler, Wilson; congruences of first and higher degrees; quadratic residues; introduction to the theory of binary quadratic forms; quadratic reciprocity; partitions.
MATH 154. Algebraic Number Theory. 3 Units.
Properties of number fields and Dedekind domains, quadratic and cyclotomic fields, applications to some classical Diophantine equations. Prerequisites: 120 and 121, especially modules over principal ideal domains and Galois theory of finite fields.

MATH 155. Analytic Number Theory. 3 Units.
Topics in analytic number theory such as the distribution of prime numbers, the prime number theorem, twin primes and Goldbach's conjecture, the theory of quadratic forms, Dirichlet's class number formula, Dirichlet's theorem on primes in arithmetic progressions, and the fifteen theorem. Prerequisite: 152, or familiarity with the Euclidean algorithm, congruences, residue classes and reduced residue classes, primitive roots, and quadratic reciprocity.

MATH 158. Basic Probability and Stochastic Processes with Engineering Applications. 3 Units.
Calculations of random variables and their distributions with applications. Review of limit theorems of probability and their application to statistical estimation and basic Monte Carlo methods. Introduction to Markov chains, random walks, Brownian motion and basic stochastic differential equations with emphasis on applications from economics, physics and engineering, such as filtering and control. Prerequisites: exposure to basic probability.
Same as: CME 298

MATH 159. Discrete Probabilistic Methods. 3 Units.
Modern discrete probabilistic methods suitable for analyzing discrete structures of the type arising in number theory, graph theory, combinatorics, computer science, information theory and molecular sequence analysis. Prerequisite: STATS 116/MATH 151 or equivalent.

MATH 161. Set Theory. 3 Units.
Informal and axiomatic set theory: sets, relations, functions, and set-theoretical operations. The Zermelo-Fraenkel axiom system and the special role of the axiom of choice and its various equivalents. Well-orderings and ordinal numbers; transfinite induction and transfinite recursion. Equinumerosity and cardinal numbers; Cantor's Alephs and cardinal arithmetic. Open problems in set theory. Prerequisite: students should be comfortable doing proofs.

MATH 162. Philosophy of Mathematics. 4 Units.
(Graduate students register for PHIL 262.) General survey of the philosophy of mathematics, focusing on epistemological issues. Includes survey of some basic concepts (proof, axiom, definition, number, set); mind-bending theorems about the limits of our current mathematical knowledge, such as Gödel's Incompleteness Theorems, and the independence of the continuum hypothesis from the current axioms of set theory; major philosophical accounts of mathematics: Logicism, Intuitionism, Hilbert's program, Quine's empiricism, Field's program, Structuralism; concluding with a discussion of Eugene Wigner's 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences'. Students won't be expected to prove theorems or complete mathematical exercises. However, includes some material of a technical nature. Prerequisite: PHIL 150 or consent of instructor.
Same as: PHIL 162, PHIL 262

MATH 163. The Greek Invention of Mathematics. 3-5 Units.
How was mathematics invented? A survey of the main creative ideas of ancient Greek mathematics. Among the issues explored are the axiomatic system of Euclid's Elements, the origins of the calculus in Greek measurements of solids and surfaces, and Archimedes' creation of mathematical physics. We will provide proofs of ancient theorems, and also learn how such theorems are even known today thanks to the recovery of ancient manuscripts.
Same as: CLASSICS 136

MATH 171. Fundamental Concepts of Analysis. 3 Units.
Recommended for Mathematics majors and required of honors Mathematics majors. Similar to 115 but altered content and more theoretical orientation. Properties of Riemann integrals, continuous functions and convergence in metric spaces; compact metric spaces, basic point set topology. Prerequisite: 61CM or 61DM or 115 or consent of the instructor. WIM.

MATH 172. Lebesgue Integration and Fourier Analysis. 3 Units.
Similar to 205A, but for undergraduate Math majors and graduate students in other disciplines. Topics include Lebesgue measure on Euclidean space, Lebesgue integration, L^p spaces, the Fourier transform, the Hardy-Littlewood maximal function and Lebesgue differentiation. Prerequisite: 171 or consent of instructor.

MATH 173. Theory of Partial Differential Equations. 3 Units.
A rigorous introduction to PDE accessible to advanced undergraduates. Elliptic, parabolic, and hyperbolic equations in many space dimensions including basic properties of solutions such as maximum principles, causality, and conservation laws. Methods include the Fourier transform as well as more classical methods. The Lebesgue integral will be used throughout, but a summary of its properties will be provided to make the course accessible to students who have not had 172 or 205A. In years when Math 173 is not offered, Math 220 is a recommended alternative (with similar content but a different emphasis). Prerequisite: 171 or equivalent.

MATH 174. Calculus of Variations. 3 Units.
An introductory course emphasizing the historical development of the theory, its connections to physics and mechanics, its independent mathematical interest, and its contacts with daily life experience. Applications to minimal surfaces and to capillary surface interfaces. Prerequisites: Math 171 or equivalent.

MATH 175. Elementary Functional Analysis. 3 Units.
Linear operators on Hilbert space. Spectral theory of compact operators; applications to integral equations. Elements of Banach space theory. Prerequisite: 115 or 171.

MATH 177. Geometric Methods in the Theory of Ordinary Differential Equations. 3 Units.
Hamiltonian systems and their geometry. First order PDE and Hamilton-Jacobi equation. Structural stability and hyperbolic dynamical systems. Completely integrable systems. Perturbation theory.

MATH 180. Introduction to Financial Mathematics. 3 Units.

MATH 193. Polya Problem Solving Seminar. 1 Unit.
Topics in mathematics and problem solving strategies with an eye towards the Putnam Competition. Topics may include parity, the pigeonhole principle, number theory, recurrence, generating functions, and probability. Students present solutions to the class. Open to anyone with an interest in mathematics.

MATH 197. Senior Honors Thesis. 1-6 Unit.
Honors math major working on senior honors thesis under an approved advisor carries out research and reading. Satisfactory written account of progress achieved during term must be submitted to advisor before term ends. May be repeated 3 times for a max of 9 units. Contact department student services specialist to enroll.
MATH 199. Independent Work. 1-3 Unit.
For math majors only. Undergraduates pursue a reading program; topics limited to those not in regular department course offerings. Credit can fulfill the elective requirement for math majors. Approval of Undergraduate Affairs Committee is required to use credit for honors majors area requirement. Contact department student services specialist to enroll.

MATH 205A. Real Analysis. 3 Units.
Basic measure theory and the theory of Lebesgue integration. Prerequisite: 171 or equivalent.

MATH 205B. Real Analysis. 3 Units.
Point set topology, basic functional analysis, Fourier series, and Fourier transform. Prerequisites: 171 and 205A or equivalent.

MATH 205C. Real Analysis. 3 Units.
Continuation of 205B.

MATH 210A. Modern Algebra I. 3 Units.
Basic commutative ring and module theory, tensor algebra, homological constructions, linear and multilinear algebra, canonical forms and Jordan decomposition. Prerequisite: 122 or equivalent.

MATH 210B. Modern Algebra II. 3 Units.
Continuation of 210A. Topics in field theory, commutative algebra, and algebraic geometry. Prerequisites: 210A, and 121 or equivalent.

MATH 210C. Lie Theory. 3 Units.
Topics in Lie groups, Lie algebras, and/or representation theory. Prerequisite: math 210B. May be repeated for credit.

MATH 215A. Algebraic Topology. 3 Units.
Topics: fundamental group and covering spaces, basics of homotopy theory, homology and cohomology (simplicial, singular, cellular, products, introduction to topological manifolds, orientations, Poincare duality. Prerequisites: 113, 120, and 171.

MATH 215B. Differential Topology. 3 Units.
Topics: Basics of differentiable manifolds (tangent spaces, vector fields, tensor fields, differential forms), embeddings, tubular neighborhoods, integration and Stokes' Theorem, deRham cohomology, intersection theory via Poincare duality, Morse theory. Prerequisite: 215A.

MATH 215C. Differential Geometry. 3 Units.
This course will be an introduction to Riemannian Geometry. Topics will include the Levi-Civita connection, Riemann curvature tensor, Ricci and scalar curvature, geodesics, parallel transport, completeness, geodesics and Jacobi fields, and comparison techniques. Prerequisites 146 or 215B.

MATH 216A. Introduction to Algebraic Geometry. 3 Units.
Algebraic curves, algebraic varieties, sheaves, cohomology, Riemann-Roch theorem. Classification of algebraic surfaces, moduli spaces, deformation theory and obstruction theory, the notion of schemes. May be repeated for credit. Prerequisites: 210ABC or equivalent.

MATH 216B. Introduction to Algebraic Geometry. 3 Units.
Continuation of 216A. May be repeated for credit.

MATH 216C. Introduction to Algebraic Geometry. 3 Units.
Continuation of 216B. May be repeated for credit.

MATH 217C. Complex Differential Geometry. 3 Units.
Complex structures, almost complex manifolds and integrability, Hermitian and Kahler metrics, connections on complex vector bundles, Chern classes and Chern-Weil theory, Hodge and Dolbeault theory, vanishing theorems, Calabi-Yau manifolds, deformation theory.

MATH 220. Partial Differential Equations of Applied Mathematics. 3 Units.
First-order partial differential equations; method of characteristics; weak solutions; elliptic, parabolic, and hyperbolic equations; Fourier transform; Fourier series; and eigenvalue problems. Prerequisite: Basic coursework in multivariable calculus and ordinary differential equations, and some prior experience with a proof-based treatment of the material as in Math 171 or Math 61CM (formerly Math 51H).

MATH 221A. Mathematical Methods of Imaging. 3 Units.
Image denoising and deblurring with optimization and partial differential equations methods. Imaging functionals based on total variation and l-1 minimization. Fast algorithms and their implementation.

MATH 221B. Mathematical Methods of Imaging. 3 Units.
Array imaging using Kirchhoff migration and beamforming, resolution theory for broad and narrow band array imaging in homogeneous media, topics in high-frequency, variable background imaging with velocity estimation, interferometric imaging methods, the role of noise and inhomogeneities, and variational problems that arise in optimizing the performance of array imaging algorithms.

MATH 222. Numerical Solution of Partial Differential Equations. 3 Units.
Hyperbolic partial differential equations: stability, convergence and qualitative properties; nonlinear hyperbolic equations and systems; combined solution methods from elliptic, parabolic, and hyperbolic problems. Examples include: Burger’s equation, Euler equations for compressible flow, Navier-Stokes equations for incompressible flow. Prerequisites: MATH 220A or CME 302.

MATH 223. Partial Differential Equations and Diffusion Processes. 3 Units.
Parabolic and elliptic partial differential equations and their relation to diffusion processes. First order equations and optimal control. Emphasis is on applications to mathematical finance. Prerequisites: MATH 131 and MATH 136/STATS 219, or equivalents.

MATH 228. Stochastic Methods in Engineering. 3 Units.
The basic limit theorems of probability theory and their application to maximum likelihood estimation. Basic Monte Carlo methods and importance sampling. Markov chains and processes, random walks, basic ergodic theory and its application to parameter estimation. Discrete time stochastic control and Bayesian filtering. Diffusion approximations, Brownian motion and an introduction to stochastic differential equations. Examples and problems from various applied areas. Prerequisites: exposure to probability and background in analysis.

MATH 228A. Probability, Stochastic Analysis and Applications. 3 Units.
The basic limit theorems of probability theory and their application to maximum likelihood estimation. Basic Monte Carlo methods and importance sampling. Markov chains and processes, random walks, basic ergodic theory and its application to parameter estimation. Discrete time stochastic control and Bayesian filtering. Diffusion approximations, Brownian motion and basic stochastic differential equations. Examples and problems from various applied areas. Prerequisites: exposure to probability and background in analysis.
MATH 230A. Theory of Probability I. 2-4 Units.
Mathematical tools: sigma algebras, measure theory, connections between coin tossing and Lebesgue measure, basic convergence theorems. Probability: independence, Borel-Cantelli lemmas, almost sure and Lp convergence, weak and strong laws of large numbers. Large deviations. Weak convergence; central limit theorems; Poisson convergence; Stein’s method. Prerequisites: 116, MATH 171.
Same as: STATS 310A

MATH 230B. Theory of Probability II. 2-3 Units.
Conditional expectations, discrete time martingales, stopping times, uniform integrability, applications to 0-1 laws, Radon-Nikodym Theorem, ruin problems, etc. Other topics as time allows selected from (i) local limit theorems, (ii) renewal theory, (iii) discrete time Markov chains, (iv) random walk theory, (v) ergodic theory. Prerequisite: 310A or MATH 230A.
Same as: STATS 310B

MATH 230C. Theory of Probability III. 2-4 Units.
Same as: STATS 310C

MATH 231A. An Introduction to Random Matrix Theory. 3 Units.
Patterns in the eigenvalue distribution of typical large matrices, which also show up in physics (energy distribution in scattering experiments), combinatorics (length of longest increasing subsequence), first passage percolation and number theory (zeros of the zeta function). Classical compact ensembles (random orthogonal matrices). The tools of determinant point processes.
Same as: STATS 351A

MATH 231C. Free Probability. 3 Units.
Background from operator theory, addition and multiplication theorems for operators, spectral properties of infinite-dimensional operators, the free additive and multiplicative convolutions of probability measures and their classical counterparts, asymptotic freeness of large random matrices, and free entropy and free dimension. Prerequisite: MATH 230B or equivalent.

MATH 232. Topics in Probability: Percollation Theory. 3 Units.
An introduction to first passage percolation and related general tools and models. Topics include early results on shape theorems and fluctuations, more modern development using hyper-contractivity, recent breakthrough regarding scaling exponents, and providing exposure to some fundamental long-standing open problems. Course prerequisite: graduate-level probability.

MATH 233A. Topics in Combinatorics. 3 Units.

MATH 233B. Topics in Combinatorics: Polyhedral Techniques in Optimization. 3 Units.
LP duality and min-max formulas; matchings, spanning trees, matroids, matroid union and intersection; packing of trees and arborescences; submodular functions, continuous extensions and optimization.

MATH 233C. Topics in Combinatorics. 3 Units.

MATH 234. Large Deviations Theory. 3 Units.
Same as: STATS 374

MATH 235A. Topics in combinatorics. 3 Units.
This advanced course in extremal combinatorics covers several major themes in the area. These include extremal combinatorics and Ramsey theory, the graph regularity method, and algebraic methods.

MATH 235B. Modern Markov Chain Theory. 3 Units.
This is a graduate-level course on the use and analysis of Markov chains. Emphasis is placed on explicit rates of convergence for chains used in applications to physics, biology, and statistics. Topics covered: basic constructions (metropolis, Gibbs sampler, data augmentation, hybrid Monte Carlo); spectral techniques (explicit diagonalization, Poincare and Cheeger bounds); functional inequalities (Nash, Sobolev, Log Sobolev); probabilistic techniques (coupling, stationary times, Harris recurrence). A variety of card shuffling processes will be studied. Central Limit and concentration.

MATH 235C. Topics in Markov Chains. 3 Units.
Classical functional inequalities (Nash, Faber-Krahn, log-Sobolev inequalities), comparison of Dirichlet forms. Random walks and isoperimetry of amenable groups (with a focus on solvable groups). Entropy, harmonic functions, and Poisson boundary (following Kaimanovich-Vershik theory).

MATH 236. Introduction to Stochastic Differential Equations. 3 Units.

MATH 237. Default and Systemic Risk. 3 Units.
Introduction to mathematical models of complex static and dynamic stochastic systems that undergo sudden regime change in response to small changes in parameters. Examples from materials science (phase transitions), power grid models, financial and banking systems. Special emphasis on mean field models and their large deviations, including computational issues. Dynamic network models of financial systems and their stability.

MATH 238. Mathematical Finance. 3 Units.
Same as: STATS 250

MATH 239. Computation and Simulation in Finance. 3 Units.
Monte Carlo, finite difference, tree, and transform methods for the numerical solution of partial differential equations in finance. Emphasis is on derivative security pricing. Prerequisite: 238 or equivalent.

MATH 243. Functions of Several Complex Variables. 3 Units.

MATH 244. Riemann Surfaces. 3 Units.
Riemann surfaces and holomorphic maps, algebraic curves, maps to projective spaces. Calculus on Riemann surfaces. Elliptic functions and integrals. Riemann-Hurwitz formula. Riemann-Roch theorem, Abel-Jacobi map. Uniformization theorem. Hyperbolic surfaces. (Suitable for advanced undergraduates.) Prerequisites: MATH 106 or MATH 116, and familiarity with surfaces equivalent to MATH 143, MATH 146, or MATH 147.

MATH 245A. Topics in Algebraic Geometry. 3 Units.
Topics of contemporary interest in algebraic geometry. May be repeated for credit.

MATH 245B. Topics in Algebraic Geometry. 3 Units.
May be repeated for credit.
MATH 245C. Topics in Algebraic Geometry. 3 Units.
May be repeated for credit.

MATH 246. Topics in number theory: L-functions. 3 Units.
The Riemann Zeta function and Dirichlet L-functions, zero-free regions and vertical distribution of the zeros, primes in arithmetic progressions, the class number problem, Hecke L-functions and Tate’s thesis, Artin L-functions and the Chebotarev density theorem, Modular forms and Maass forms.

Prerequisites: Algebraic Number Theory.

MATH 248. Introduction to Ergodic Theory. 3 Units.
Topics may include 1) subadditive and multiplicative ergodic theorems, 2) notions of mixing, weak mixing, spectral theory, 3) metric and topological entropy of dynamical systems, 4) measures of maximal entropy.

Prerequisites: Solid background in “Measure and Integration” (Math 205A) and some functional analysis, including Riesz representation and Hahn-Banach theorem (Math 205B).

MATH 249A. Topics in number theory. 3 Units.
Topics of contemporary interest in number theory. May be repeated for credit.

MATH 249B. Topics in Number Theory. 3 Units.

MATH 249C. Topics in Number Theory. 3 Units.

MATH 256A. Partial Differential Equations. 3 Units.
The theory of linear and nonlinear partial differential equations, beginning with linear theory involving use of Fourier transform and Sobolev spaces. Topics: Schauder and L2 estimates for elliptic and parabolic equations; De Giorgi-Nash-Moser theory for elliptic equations; nonlinear equations such as the minimal surface equation, geometric flow problems, and nonlinear hyperbolic equations.

MATH 256B. Partial Differential Equations. 3 Units.
Continuation of 256A.

MATH 257A. Symplectic Geometry and Topology. 3 Units.
Linear symplectic geometry and linear Hamiltonian systems. Symplectic manifolds and their Lagrangian submanifolds, local properties. Symplectic geometry and mechanics. Contact geometry and contact manifolds. Relations between symplectic and contact manifolds. Hamiltonian systems with symmetries. Momentum map and its properties. May be repeated for credit.

MATH 257B. Symplectic Geometry and Topology. 3 Units.
Continuation of 257A. May be repeated for credit.

MATH 257C. Symplectic Geometry and Topology. 3 Units.
Continuation of 257B. May be repeated for credit.

MATH 258. Topics in Geometric Analysis. 3 Units.
May be repeated for credit.

MATH 262. Applied Fourier Analysis and Elements of Modern Signal Processing. 3 Units.
Introduction to the mathematics of the Fourier transform and how it arises in a number of imaging problems. Mathematical topics include the Fourier transform, the Plancherel theorem, Fourier series, the Shannon sampling theorem, the discrete Fourier transform, and the spectral representation of stationary stochastic processes. Computational topics include fast Fourier transforms (FFT) and nonuniform FFTs. Applications include Fourier imaging (the theory of diffraction, computed tomography, and magnetic resonance imaging) and the theory of compressive sensing.

Same as: CME 372

MATH 263A. Algebraic Combinatorics and Symmetric Functions. 3 Units.
Symmetric function theory unifies large parts of combinatorics. Theorems about permutations, partitions, and graphs now follow in a unified way. Topics: The usual bases (monomial, elementary, complete, and power sums). Schur functions. Representation theory of the symmetric group. Littlewood-Richardson rule, quasi-symmetric functions, combinatorial Hopf algebras, introduction to Macdonald polynomials. Throughout, emphasis is placed on applications (e.g. to card shuffling and random matrix theory). Prerequisite: 210A and 210B, or equivalent.

MATH 263B. Crystal Bases: Representations and Combinatorics. 3 Units.
Crystal Bases are combinatorial analogs of representation theory of Lie groups. We will explore different aspects of these analogies and develop rigorous purely combinatorial foundations.

MATH 263C. Topics in Representation Theory. 3 Units.
May be repeated for credit.

MATH 269. Topics in symplectic geometry. 3 Units.
May be repeated for credit.

MATH 270. Geometry and Topology of Complex Manifolds. 3 Units.
Complex manifolds, Kahler manifolds, curvature, Hodge theory, Lefschetz theorem, Kahler-Einstein equation, Hermitian-Einstein equations, deformation of complex structures. May be repeated for credit.

MATH 271. The H-Principle. 3 Units.

MATH 272. Topics in Partial Differential Equations. 3 Units.

MATH 273. Topics in Mathematical Physics. 3 Units.
Covers a list of topics in mathematical physics. The specific topics may vary from year to year, depending on the instructor's discretion. Background in graduate level probability theory and analysis is desirable. Same as: STATS 359

MATH 280. Evolution Equations in Differential Geometry. 3 Units.

MATH 282A. Low Dimensional Topology. 3 Units.
The theory of surfaces and 3-manifolds. Curves on surfaces, the classification of diffeomorphisms of surfaces, and Teichmüller space. The mapping class group and the braid group. Knot theory, including knot invariants. Decomposition of 3-manifolds: triangulations, Heegaard splittings, Dehn surgery. Loop theorem, sphere theorem, incompressible surfaces. Geometric structures, particularly hyperbolic structures on surfaces and 3-manifolds. May be repeated for credit up to 6 total units.

MATH 282B. Homotopy Theory. 3 Units.
Homotopy groups, fibrations, spectral sequences, simplicial methods, Dold-Thom theorem, models for loop spaces, homotopy limits and colimits, stable homotopy theory. May be repeated for credit up to 6 total units.

MATH 282C. Fiber Bundles and Cobordism. 3 Units.

MATH 283. Topics in Algebraic and Geometric Topology. 3 Units.
May be repeated for credit.

MATH 283A. Topics in Topology. 3 Units.
MATH 284. Topics in Geometric Topology. 3 Units.
Incompressible surfaces, irreducible manifolds, prime decomposition, Morse theory, Heegaard diagrams, Heegaard splittings, the Thurston norm, sutured manifold theory, Heegaard Floer homology, sutured Floer homology.

MATH 284A. Geometry and Topology in Dimension 3. 3 Units.
The Poincare conjecture and the uniformization of 3-manifolds. May be repeated for credit.

MATH 284B. Geometry and Topology in Dimension 3. 3 Units.
The Poincare conjecture and the uniformization of 3-manifolds. May be repeated for credit.

MATH 286. Topics in Differential Geometry. 3 Units.
May be repeated for credit.

MATH 298. Graduate Practical Training. 1 Unit.
Only for mathematics graduate students. Students obtain employment in a relevant industrial or research activity to enhance their professional experience. Students submit a concise report detailing work activities, problems worked on, and key results. May be repeated for credit up to 3 units. Prerequisite: qualified offer of employment and consent of department. Prior approval by Math Department is required; you must contact the Math Department's Student Services staff for instructions before being granted permission to enroll.

MATH 301. Advanced Topics in Convex Optimization. 3 Units.
Modern developments in convex optimization: semidefinite programming; novel and efficient first-order algorithms for smooth and nonsmooth convex optimization. Emphasis on numerical methods suitable for large scale problems arising in science and engineering. Prerequisites: convex optimization (EE 364), linear algebra (Math 104), numerical linear algebra (CME 302); background in probability, statistics, real analysis and numerical optimization.
Same as: CME 375

MATH 305. Applied mathematics through toys and magic. 3 Units.
This course is a series of case-studies in doing applied mathematics on surprising phenomena we notice in daily life. Almost every class will show demos of these phenomena (toys and magic) and suggest open projects. The topics range over a great variety and cut across areas traditionally pigeonholed as physics, biology, engineering, computer science, mathematics but, instead of developing sophisticated mathematics on simple material, our aim is to extract simple mathematical understanding from sophisticated material which, at first, we may not yet know how to pigeonhole. In each class I will try to make the discussion self-contained and to give everybody something to take home, regardless of the background.

MATH 355. Graduate Teaching Seminar. 1 Unit.
Required of and limited to first-year Mathematics graduate students.

MATH 360. Advanced Reading and Research. 1-10 Unit.

MATH 382. Qualifying Examination Seminar. 1-3 Unit.

MATH 391. Research Seminar in Logic. 1-3 Unit.
Contemporary work. May be repeated a total of three times for credit. Same as: PHIL 391

MATH 802. TGR Dissertation. 0 Units.