MATH 101. Math Discovery Lab. 3 Units.
MDL is a discovery-based project course in mathematics. Students work independently in small groups to explore open-ended mathematical problems and discover original mathematics. Students formulate conjectures and hypotheses; test predictions by computation, simulation, or pure thought; and present their results to classmates. No lecture component; in-class meetings reserved for student presentations, attendance mandatory. Admission is by application: http://math101.stanford.edu. Motivated students with any level of mathematical background are encouraged to apply. WIM.

MATH 104. Applied Matrix Theory. 3 Units.
Linear algebra for applications in science and engineering: orthogonality, projections, spectral theory for symmetric matrices, the singular value decomposition, the QR decomposition, least-squares, the condition number of a matrix, algorithms for solving linear systems. MATH 113 offers a more theoretical treatment of linear algebra. MATH 104 and EE 103/CME 103 cover complementary topics in applied linear algebra. The focus of MATH 104 is on algorithms and concepts; the focus of EE 103 is on a few linear algebra concepts, and many applications. Prerequisites: MATH 51 and programming experience on par with CS 106.

MATH 106. Functions of a Complex Variable. 3 Units.
Complex numbers, analytic functions, Cauchy-Riemann equations, complex integration, Cauchy integral formula, residues, elementary conformal mappings. (Math 116 offers a more theoretical treatment.) Prerequisite: 52.

MATH 107. Graph Theory. 3 Units.
An introductory course in graph theory establishing fundamental concepts and results in variety of topics. Topics include: basic notions, connectivity, cycles, matchings, planar graphs, graph coloring, matrix-tree theorem, conditions for hamiltonicity, Kuratowski's theorem, Ramsey and Turan-type theorem. Prerequisites: 51 or equivalent and some familiarity with proofs is required.

MATH 108. Introduction to Combinatorics and Its Applications. 3 Units.
Topics: graphs, trees (Cayley's Theorem, application to phylogony), eigenvalues, basic enumeration (permutations, Stirling and Bell numbers), recurrences, generating functions, basic asymptotics. Prerequisites: 51 or equivalent.

MATH 109. Applied Group Theory. 3 Units.
Applications of the theory of groups. Topics: elements of group theory, groups of symmetries, matrix groups, group actions, and applications to combinatorics and computing. Applications: rotational symmetry groups, the study of the Platonic solids, crystallographic groups and their applications in chemistry and physics. Honors math majors and students who intend to do graduate work in mathematics should take 120. WIM.

MATH 110. Applied Number Theory and Field Theory. 3 Units.
Number theory and its applications to modern cryptography. Topics: congruences, finite fields, primality testing and factorization, public key cryptography, error correcting codes, and elliptic curves, emphasizing algorithms. WIM.

MATH 113. Linear Algebra and Matrix Theory. 3 Units.
Algebraic properties of matrices and their interpretation in geometric terms. The relationship between the algebraic and geometric points of view and matters fundamental to the study and solution of linear equations. Topics: linear equations, vector spaces, linear dependence, bases and coordinate systems; linear transformations and matrices; similarity; eigenvectors and eigenvalues; diagonalization. (Math 104 offers a more application-oriented treatment.).

MATH 114. Introduction to Scientific Computing. 3 Units.
Introduction to Scientific Computing Numerical computation for mathematical, computational, physical sciences and engineering: error analysis, floating-point arithmetic, nonlinear equations, numerical solution of systems of algebraic equations, banded matrices, least squares, unconstrained optimization, polynomial interpolation, numerical differentiation and integration, numerical solution of ordinary differential equations, truncation error, numerical stability for time dependent problems and stiffness. Implementation of numerical methods in MATLAB programming assignments. Prerequisites: MATH 51, 52, 53; prior programming experience (MATLAB or other language at level of CS 106A or higher).
Same as: CME 108

MATH 115. Functions of a Real Variable. 3 Units.
The development of real analysis in Euclidean space: sequences and series, limits, continuous functions, derivatives, integrals. Basic point set topology. Honors math majors and students who intend to do graduate work in mathematics should take 171. Prerequisite: 21.

MATH 116. Complex Analysis. 3 Units.
Analytic functions, Cauchy integral formula, power series and Laurent series, calculus of residues and applications, conformal mapping, analytic continuation, introduction to Riemann surfaces, Fourier series and integrals. (Math 106 offers a less theoretical treatment.) Prerequisites: 52, and 115 or 171.

MATH 118. Mathematics of Computation. 3 Units.
Notions of analysis and algorithms central to modern scientific computing: continuous and discrete Fourier expansions, the fast Fourier transform, orthogonal polynomials, interpolation, quadrature, numerical differentiation, analysis and discretization of initial-value and boundary-value ODE, finite and spectral elements. Prerequisites: MATH 51 and 53.

MATH 120. Groups and Rings. 3 Units.
Recommended for Mathematics majors and required of honors Mathematics majors. Similar to 109 but altered content and more theoretical orientation. Groups acting on sets, examples of finite groups, Sylow theorems, solvable and simple groups. Fields, rings, and ideals; polynomial rings over a field; PID and non-PID. Unique factorization domains. WIM.

MATH 121. Galois Theory. 3 Units.
Field of fractions, splitting fields, separability, finite fields. Galois groups, Galois correspondence, examples and applications. Prerequisite: Math 120 and (also recommended) 113.

MATH 122. Modules and Group Representations. 3 Units.
Modules over PID. Tensor products over fields. Group representations and group rings. Maschke's theorem and character theory. Character tables, construction of representations. Prerequisite: Math 120. Also recommended: 113.

MATH 131P. Partial Differential Equations. 3 Units.
An introduction to PDE; particularly suitable for non-Math majors. Topics include physical examples of PDE's, method of characteristics, D'Alembert's formula, maximum principles, heat kernel, Duhamel's principle, separation of variables, Fourier series, Harmonic functions, Bessel functions, spherical harmonics. Students who have taken MATH 171 should consider taking MATH 173 rather than 131P. Prerequisite: 53.

MATH 136. Stochastic Processes. 3 Units.
Same as: STATS 219
MATH 137. Mathematical Methods of Classical Mechanics. 3 Units.
Oscillations. Rigid bodies. Introduction to symplectic geometry.
Hamiltonian formalism. Legendre transform. Variational principles.
Geometric optics. Introduction to the theory of integrable systems.
Prerequisites: 51, 52, 53, or 61CM, 62CM, 63CM.

MATH 138. Celestial Mechanics. 3 Units.
Mathematically rigorous introduction to the classical N-body problem: the
motion of N particles evolving according to Newton's law. Topics include:
the Kepler problem and its symmetries; other central force problems;
conservation theorems; variational methods; Hamilton-Jacobi theory;
the role of equilibrium points and stability; and symplectic methods.
Prerequisites: 53, and 115 or 171.

MATH 142. Hyperbolic Geometry. 3 Units.
An introductory course in hyperbolic geometry. Topics may include:
different models of hyperbolic geometry, hyperbolic area and geodesics,
Isometries and Mobius transformations, conformal maps, Fuchsian
groups, Farey tessellation, hyperbolic structures on surfaces and three
manifolds, limit sets. Prerequisites: some familiarity with the basic
concepts of differential geometry and the topology of surfaces and
manifolds is strongly recommended.

MATH 143. Differential Geometry. 3 Units.
Geometry of curves and surfaces in three-space and higher dimensional
manifolds. Parallel transport, curvature, and geodesics. Surfaces with
constant curvature. Minimal surfaces.

MATH 145. Algebraic Geometry. 3 Units.
An introduction to the methods and concepts of algebraic geometry. The
point of view and content will vary over time, but include: affine varieties,
Hilbert basis theorem and Nullstellensatz, projective varieties, algebraic
curves. Required: 120. Strongly recommended: additional mathematical
maturity via further basic background with fields, point-set topology, or
manifolds.

MATH 146. Analysis on Manifolds. 3 Units.
Differentiable manifolds, tangent space, submanifolds, implicit function
theorem, differential forms, vector and tensor fields. Frobenius' theorem,
DeRham theory. Prerequisite: 62CM or 52 and familiarity with linear
algebra and analysis arguments at the level of 113 and 115 respectively.

MATH 147. Differential Topology. 3 Units.
Smooth manifolds, transversality, Sard’s theorem, embeddings, degree
of a map, Borsuk-Ulam theorem, Hopf degree theorem, Jordan curve
theorem. Prerequisite: 115 or 171.

MATH 148. Algebraic Topology. 3 Units.
Fundamental group, covering spaces, Euler characteristic, homology;
classification of surfaces, knots. Prerequisite: 109 or 120.

MATH 151. Introduction to Probability Theory. 3 Units.
Counting: axioms of probability; conditioning and independence;
expectation and variance; discrete and continuous random variables and
distributions; joint distributions and dependence; central limit theorem
and laws of large numbers. Prerequisite: 52 or consent of instructor.

MATH 152. Elementary Theory of Numbers. 3 Units.
Euclid's algorithm, fundamental theorems on divisibility; prime numbers;
congruence of numbers; theorems of Fermat, Euler, Wilson; congruences
of first and higher degrees; quadratic residues; introduction to the theory
of binary quadratic forms; quadratic reciprocity; partitions.

MATH 154. Algebraic Number Theory. 3 Units.
Properties of number fields and Dedekind domains, quadratic and
cyclotomic fields, applications to some classical Diophantine equations.
Prerequisites: 120 and 121, especially modules over principal ideal
domains and Galois theory of finite fields.

MATH 155. Analytic Number Theory. 3 Units.
Topics in analytic number theory such as the distribution of prime
numbers, the prime number theorem, twin primes and Goldbach's
conjecture, the theory of quadratic forms, Dirichlet's class number
formula, Dirichlet's theorem on primes in arithmetic progressions, and
the fifteen theorem. Prerequisite: 152, or familiarity with the Euclidean
algorithm, congruences, residue classes and reduced residue classes,
primitive roots, and quadratic reciprocity.

MATH 158. Basic Probability and Stochastic Processes with Engineering
Applications. 3 Units.
Calculus of random variables and their distributions with applications.
Review of limit theorems of probability and their application to statistical
estimation and basic Monte Carlo methods. Introduction to Markov
chains, random walks, Brownian motion and basic stochastic differential
equations with emphasis on applications from economics, physics and
engineering, such as filtering and control. Prerequisites: exposure to
basic probability.
Same as: CME 298

MATH 159. Discrete Probabilistic Methods. 3 Units.
Modern discrete probabilistic methods suitable for analyzing
discrete structures of the type arising in number theory, graph theory;
combinatorics, computer science, information theory and molecular
sequence analysis. Prerequisite: STATS 116/MATH 151 or equivalent.
Typically in alternating years.

MATH 161. Set Theory. 3 Units.
Informal and axiomatic set theory: sets, relations, functions, and set-
theoretical operations. The Zermelo-Fraenkel axiom system and the
special role of the axiom of choice and its various equivalents. Well-
orderings and ordinal numbers; transfinite induction and transfinite
recursion. Equinumerosity and cardinal numbers; Cantor's Alephs and
cardinal arithmetic. Open problems in set theory. Prerequisite: students
should be comfortable doing proofs.

MATH 162. Philosophy of Mathematics. 4 Units.
Mathematics is a very peculiar human activity. It delivers a type of
knowledge that is particularly stable, often conceived as a priori and
necessary. Moreover, this knowledge is about abstract entities, which
seem to have no connection to us, spatio-temporal creatures, and yet
it plays a crucial role in our scientific endeavors. Many philosophical
questions emerge naturally: What is the nature of mathematical objects?
How can we learn anything about them? Where does the stability of
mathematics come from? What is the significance of results showing
the limits of such knowledge, such as Gödel’s incompleteness theorem?
The first part of the course will survey traditional approaches to
philosophy of mathematics ("the big isms") and consider the viability of
their answers to some of the previous questions: logicism, intuitionism,
Hilbert’s program, empiricism, fictionalism, and structuralism. The
second part will focus on philosophical issues emerging from the
actual practice of mathematics. We will tackle questions such as:
Why do mathematicians re-prove the same theorems? What is the role
of visualization in mathematics? How can mathematical knowledge
be effective in natural science? To conclude, we will explore the
aesthetic dimension of mathematics, focusing on mathematical beauty.
Prerequisite: PHIL150 or consent of instructor.
Same as: PHIL 162

MATH 171. Fundamental Concepts of Analysis. 3 Units.
Recommended for Mathematics majors and required of honors
Mathematics majors. Similar to 115 but altered content and more
theoretical orientation. Properties of Riemann integrals, continuous
functions and convergence in metric spaces; compact metric spaces,
basic point set topology. Prerequisite: 61CM or 61DM or 115 or consent
of the instructor. WIM.
MATH 172. Lebesgue Integration and Fourier Analysis. 3 Units.
Similar to 205A, but for undergraduate Math majors and graduate students in other disciplines. Topics include Lebesgue measure on Euclidean space, Lebesgue integration, $L^p$ spaces, the Fourier transform, the Hardy-Littlewood maximal function and Lebesgue differentiation. Prerequisite: 171 or consent of instructor.

MATH 173. Theory of Partial Differential Equations. 3 Units.
A rigorous introduction to PDE accessible to advanced undergraduates. Elliptic, parabolic, and hyperbolic equations in many space dimensions including basic properties of solutions such as maximum principles, causality, and conservation laws. Methods include the Fourier transform as well as more classical methods. The Lebesgue integral will be used throughout, but a summary of its properties will be provided to make the course accessible to students who have not had 172 or 205A. In years when Math 173 is not offered, Math 220 is recommended alternative (with similar content but a different emphasis). Prerequisite: 171 or equivalent.

MATH 174. Calculus of Variations. 3 Units.
An introductory course emphasizing the historical development of the theory, its connections to physics and mechanics, its independent mathematical interest, and its contacts with daily life experience. Applications to minimal surfaces and to capillary surface interfaces. Prerequisites: Math 171 or equivalent.

MATH 175. Elementary Functional Analysis. 3 Units.
Linear operators on Hilbert space. Spectral theory of compact operators; applications to integral equations. Elements of Banach space theory. Prerequisite: 115 or 171.

MATH 176. Analytic Functions of One and Several Variables. 3 Units.
Study of complex functions, analyticity, power series, integration, and residues. Prerequisite: 171 or 205A.

MATH 177. Geometric Methods in the Theory of Ordinary Differential Equations. 3 Units.
Hamiltonian systems and their geometry. First order PDE and Hamilton-Jacobi equation. Structural stability and hyperbolic dynamical systems. Completely integrable systems. Perturbation theory.

MATH 19. Calculus. 3 Units.
Introduction to differential calculus of functions of one variable. Review of elementary functions (including exponentials and logarithms), limits, rates of change, the derivative and its properties, applications of the derivative. Prerequisites: trigonometry, advanced algebra, and analysis of elementary functions (including exponentials and logarithms). You must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

MATH 193. Polya Problem Solving Seminar. 1 Unit.
Topics in mathematics and problem solving strategies with an eye towards the Putnam Competition. Topics may include parity, the pigeonhole principle, number theory, recurrence, generating functions, and probability. Students present solutions to the class. Open to anyone with an interest in mathematics.

MATH 197. Senior Honors Thesis. 1-6 Unit.
Honors math major working on senior honors thesis under an approved advisor carries out research and reading. Satisfactory written account of progress achieved during term must be submitted to advisor before term ends. May be repeated 3 times for a max of 9 units. Contact department student services specialist to enroll.

MATH 198. Practical Training. 1 Unit.
Only for undergraduate students majoring in mathematics. Students obtain employment in a relevant industrial or research activity to enhance their professional experience. Students submit a concise report detailing work activities, problems worked on, and key results. May be repeated for credit up to 3 units. Prerequisite: qualified offer of employment and consent of department. Prior approval by Math Department is required; you must contact the Math Department’s Student Services staff for instructions before being granted permission to enroll.

MATH 199. Independent Work. 1-3 Unit.
For math majors only. Undergraduates pursue a reading program; topics limited to those not in regular department course offerings. Credit can fulfill the elective requirement for math majors. Approval of Undergraduate Affairs Committee is required to use credit for honors majors area requirement. Contact department student services specialist to enroll.

MATH 20. Calculus. 3 Units.
The definite integral, Riemann sums, antiderivatives, the Fundamental Theorem of Calculus, and the Mean Value Theorem for integrals. Integration by substitution and by parts. Area between curves, and volume by slices, washers, and shells. Initial-value problems, exponential and logistic models, direction fields, and parametric curves. Prerequisite: Math 19 or equivalent. If you have not previously taken a calculus course at Stanford then you must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

MATH 205A. Real Analysis. 3 Units.
Basic measure theory and the theory of Lebesgue integration. Prerequisite: 171 or equivalent.

MATH 205B. Real Analysis. 3 Units.
Point set topology, basic functional analysis, Fourier series, and Fourier transform. Prerequisites: 171 and 205A or equivalent.

MATH 205C. Real Analysis. 3 Units.
Continuation of 205B.

MATH 21. Calculus. 4 Units.
Review of limit rules. Sequences, functions, limits at infinity, and comparison of growth of functions. Review of integration rules, integrating rational functions, and improper integrals. Infinite series, special examples, convergence and divergence tests (limit comparison and alternating series tests). Power series and interval of convergence, Taylor polynomials, Taylor series and applications. Prerequisite: Math 20 or equivalent. If you have not previously taken a calculus course at Stanford then you must have taken the math placement diagnostic (offered through the Math Department website) in order to register for this course.

MATH 210A. Modern Algebra I. 3 Units.
Basic commutative ring and module theory, tensor algebra, homological constructions, linear and multilinear algebra, canonical forms and Jordan decomposition. Prerequisite: 122 or equivalent.

MATH 210B. Modern Algebra II. 3 Units.
Continuation of 210A. Topics in field theory, commutative algebra, and algebraic geometry. Prerequisites: 210A, and 121 or equivalent.

MATH 210C. Lie Theory. 3 Units.
Topics in Lie groups, Lie algebras, and/or representation theory. Prerequisite: math 210B. May be repeated for credit.

MATH 215A. Algebraic Topology. 3 Units.
Topics: fundamental group and covering spaces, basics of homotopy theory, homology and cohomology (simplicial, singular, cellular), products, introduction to topological manifolds, orientations, Poincare duality. Prerequisites: 113, 120, and 171.

MATH 215B. Differential Topology. 3 Units.
Topics: Basics of differentiable manifolds (tangent spaces, vector fields, tensor fields, differential forms), embeddings, tubular neighborhoods, integration and Stokes theorem, deRham cohomology, intersection theory via Poincare duality, Morse theory. Prerequisite: 215A.

MATH 215C. Differential Geometry. 3 Units.
This course will be an introduction to Riemannian Geometry. Topics will include the Levi-Civita connection, Riemann curvature tensor, Ricci and scalar curvature, geodesics, parallel transport, completeness, geodesics and Jacobi fields, and comparison techniques. Prerequisites 146 or 215B.
MATH 216A. Introduction to Algebraic Geometry. 3 Units.
Algebraic curves, algebraic varieties, sheaves, cohomology, Riemann-Roch theorem. Classification of algebraic surfaces, moduli spaces, deformation theory and obstruction theory, the notion of schemes. May be repeated for credit. Prerequisites: 210ABC or equivalent.

MATH 216B. Introduction to Algebraic Geometry. 3 Units.
Continuation of 216A. May be repeated for credit.

MATH 216C. Introduction to Algebraic Geometry. 3 Units.
Continuation of 216B. May be repeated for credit.

MATH 217C. Complex Differential Geometry. 3 Units.
Complex structures, almost complex manifolds and integrability, Hermitian and Kahler metrics, connections on complex vector bundles, Chern classes and Chern-Weil theory, Hodge and Dolbeault theory, vanishing theorems, Calabi-Yau manifolds, deformation theory.

MATH 21A. Calculus, ACE. 5 Units.
Students attend MATH 21 lectures with different recitation sessions: two hours per week instead of one, emphasizing engineering applications. Prerequisite: application; see https://web.stanford.edu/dept/soe/osa/ace.fb.

MATH 220. Partial Differential Equations of Applied Mathematics. 3 Units.
First-order partial differential equations; method of characteristics; weak solutions; elliptic, parabolic, and hyperbolic equations; Fourier transform; Fourier series; and eigenvalue problems. Prerequisite: Basic coursework in multivariable calculus and ordinary differential equations, and some prior experience with a proof-based treatment of the material as in Math 171 or Math 61CM (formerly Math 51H). Same as: CME 303

MATH 221A. Mathematical Methods of Imaging. 3 Units.
Image denoising and deblurring with optimization and partial differential equations methods. Imaging functionals based on total variation and l1 minimization. Fast algorithms and their implementation. Same as: CME 321A

MATH 221B. Mathematical Methods of Imaging. 3 Units.
Array imaging using Kirchhoff migration and beamforming, resolution theory for broad and narrow band array imaging in homogeneous media, topics in high-frequency, variable background imaging with velocity estimation, interferometric imaging methods, the role of noise and inhomogeneities, and variational problems that arise in optimizing the performance of array imaging algorithms. Same as: CME 321B

MATH 226. Numerical Solution of Partial Differential Equations. 3 Units.
Hyperbolic partial differential equations: stability, convergence and qualitative properties; nonlinear hyperbolic equations and systems; combined solution methods from elliptic, parabolic, and hyperbolic problems. Examples include: Burger's equation, Euler equations for compressible flow, Navier-Stokes equations for incompressible. Prerequisites: MATH 220A or CME 302. Same as: CME 306

MATH 227. Partial Differential Equations and Diffusion Processes. 3 Units.
Parabolic and elliptic partial differential equations and their relation to diffusion processes. First order equations and optimal control. Emphasis is on applications to mathematical finance. Prerequisites: MATH 136/STATS 219 (or equivalents) and MATH 131P + MATH 115/171 or MATH 173 or MATH 220.

MATH 228. Stochastic Methods in Engineering. 3 Units.
The basic limit theorems of probability theory and their application to maximum likelihood estimation. Basic Monte Carlo methods and importance sampling. Markov chains and processes, random walks, basic ergodic theory and its application to parameter estimation. Discrete time stochastic control and Bayesian filtering. Diffusion approximations, Brownian motion and an introduction to stochastic differential equations. Examples and problems from various applied areas. Prerequisites: exposure to probability and background in analysis. Same as: CME 308, MS&E 324

MATH 229. Probability, Stochastic Analysis and Applications. 3 Units.
The basic limit theorems of probability theory and their application to maximum likelihood estimation. Basic Monte Carlo methods and importance sampling. Markov chains and processes, random walks, basic ergodic theory and its application to parameter estimation. Discrete time stochastic control and Bayesian filtering. Diffusion approximations, Brownian motion and basic stochastic differential equations. Examples and problems from various applied areas. Prerequisites: exposure to probability and background in analysis.

MATH 230A. Theory of Probability I. 2-4 Units.
Mathematical tools: sigma algebras, measure theory, connections between coin tossing and Lebesgue measure, basic convergence theorems. Probability: independence, Borel-Cantelli lemmas, almost sure and Lp convergence, weak and strong laws of large numbers. Large deviations. Weak convergence; central limit theorems; Poisson convergence; Stein’s method. Prerequisites: 116, MATH 171. Same as: STATS 310A

MATH 230B. Theory of Probability II. 2-3 Units.
Conditional expectations, discrete time martingales, stopping times, uniform integrability, applications to 0-1 laws, Radon-Nikodym Theorem, ruin problems, etc. Other topics as time allows selected from (i) local limit theorems, (ii) renewal theory, (iii) discrete time Markov chains, (iv) random walk theory, (v) ergodic theory. Prerequisite: 310A or MATH 230A. Same as: STATS 310B

MATH 230C. Theory of Probability III. 2-4 Units.
Same as: STATS 310C

MATH 231. Mathematics and Statistics of Gambling. 3 Units.
Probability and statistics are founded on the study of games of chance. Nowadays, gambling (in casinos, sports and the Internet) is a huge business. This course addresses practical and theoretical aspects. Topics covered: mathematics of basic random phenomena (physics of coin tossing and roulette, analysis of various methods of shuffling cards), odds in popular games, card counting, optimal tournament play, practical problems of random number generation. Prerequisites: Statistics 116 and 200.
Same as: STATS 334

MATH 231A. An Introduction to Random Matrix Theory. 3 Units.
Patterns in the eigenvalue distribution of typical large matrices, which also show up in physics (energy distribution in scattering experiments), combinatorics (length of longest increasing subsequence), first passage percolation and number theory (zeros of the zeta function). Classical compact ensembles (random orthogonal matrices). The tools of determinental point processes. Same as: STATS 351A
MATH 231C. Free Probability. 3 Units.
Background from operator theory, addition and multiplication theorems for operators, spectral properties of infinite-dimensional operators, the free additive and multiplicative convolutions of probability measures and their classical counterparts, asymptotic freeness of large random matrices, and free entropy and free dimension. Prerequisite: STATS 310B or equivalent.

MATH 232. Topics in Probability: Percolation Theory. 3 Units.
An introduction to first passage percolation and related general tools and models. Topics include early results on shape theorems and fluctuations, more modern development using hyper-contractivity, recent breakthrough regarding scaling exponents, and providing exposure to some fundamental long-standing open problems. Course prerequisite: graduate-level probability.

MATH 233A. Topics in Combinatorics. 3 Units.
Geometry of polynomials and non-constructive proofs in combinatorics: The independence polynomial, the Lovasz Local Lemma and Shearer's Lemma. Real-rooted polynomials, stable polynomials, Ramanujan graphs and the Kadison-Singer problem. Strongly Rayleigh measures and negative dependence. Applications in algorithms.

MATH 233B. Topics in Combinatorics: Polyhedral Techniques in Optimization. 3 Units.
LP duality and min-max formulas; matchings, spanning trees, matroids, matroid union and intersection; packing of trees and arborescences; submodular functions, continuous extensions and optimization.

MATH 233C. Topics in Combinatorics. 3 Units.

MATH 234. Large Deviations Theory. 3 Units.
Same as: STATS 374

MATH 235A. Topics in combinatorics. 3 Units.
This advanced course in extremal combinatorics covers several major themes in the area. These include extremal combinatorics and Ramsey theory, the graph regularity method, and algebraic methods.

MATH 235B. Modern Markov Chain Theory. 3 Units.
This is a graduate-level course on the use and analysis of Markov chains. Emphasis is placed on explicit rates of convergence for chains used in applications to physics, biology, and statistics. Topics covered: basic constructions (metropolis, Gibbs sampler, data augmentation, hybrid Monte Carlo); spectral techniques (explicit diagonalization, Poincaré, and Cheeger bounds); functional inequalities (Nash, Sobolev, Log Sobolev); probabilistic techniques (coupling, stationary times, Harris recurrence). A variety of card shuffling processes will be studied. Central Limit and concentration.

MATH 235C. Topics in Markov Chains. 3 Units.
Classical functional inequalities (Nash, Faber-Krahn, log-Sobolev inequalities), comparison of Dirichlet forms. Random walks and isoperimetry of amenable groups (with a focus on solvable groups). Entropy, harmonic functions, and Poisson boundary (following Kaimanovich-Vershik theory).

MATH 236. Introduction to Stochastic Differential Equations. 3 Units.

MATH 237. Default and Systemic Risk. 3 Units.
Introduction to mathematical models of complex static and dynamic stochastic systems that undergo sudden regime change in response to small changes in parameters. Examples from materials science (phase transitions), power grid models, financial and banking systems. Special emphasis on mean field models and their large deviations, including computational issues. Dynamic network models of financial systems and their stability.

MATH 237A. Topics in Financial Math: Market microstructure and trading algorithms. 3 Units.
Introduction to market microstructure theory, including optimal limit order and market trading models. Random matrix theory covariance models and their application to portfolio theory. Statistical arbitrage algorithms.

MATH 238. Mathematical Finance. 3 Units.
Same as: STATS 250

MATH 239. Computation and Simulation in Finance. 3 Units.
Monte Carlo, finite difference, tree, and transform methods for the numerical solution of partial differential equations in finance. Emphasis is on derivative security pricing. Prerequisite: 238 or equivalent.

MATH 243. Functions of Several Complex Variables. 3 Units.

MATH 244. Riemann Surfaces. 3 Units.
Riemann surfaces and holomorphic maps, algebraic curves, maps to projective spaces. Calculus on Riemann surfaces. Elliptic functions and integrals. Riemann-Hurwitz formula, Riemann-Roch theorem, Abel-Jacobi map. Uniformization theorem. Hyperbolic surfaces. (Suitable for advanced undergraduates.) Prerequisites: MATH 106 or MATH 116, and familiarity with surfaces equivalent to MATH 143, MATH 146, or MATH 147.

MATH 245A. Topics in Algebraic Geometry. 3 Units.
Topics of contemporary interest in algebraic geometry. May be repeated for credit.

MATH 245B. Topics in Algebraic Geometry. 3 Units.
May be repeated for credit.

MATH 245C. Topics in Algebraic Geometry. 3 Units.
May be repeated for credit.

MATH 246. Topics in number theory: L-functions. 3 Units.
The Riemann Zeta function and Dirichlet L-functions, zero-free regions and vertical distribution of the zeros, primes in arithmetic progressions, the class number problem, Hecke L-functions and Tate's thesis, Artin L-functions and the Chebotarev density theorem, Modular forms and Maass forms.nnPrerequisites: Algebraic Number Theory.

MATH 248. Introduction to Ergodic Theory. 3 Units.
Topics may include 1) subadditive and multiplicative ergodic theorems, 2) notions of mixing, weak mixing, spectral theory, 3) metric and topological entropy of dynamical systems, 4) measures of maximal entropy. Prerequisites: Solid background in "Measure and Integration" (Math 205A) and some functional analysis, including Riesz representation theorem and Hahn-Banach theorem (Math 205B).

MATH 249A. Topics in number theory. 3 Units.
Topics of contemporary interest in number theory. May be repeated for credit.

MATH 249B. Topics in Number Theory. 3 Units.

MATH 249C. Topics in Number Theory. 3 Units.

MATH 256A. Partial Differential Equations. 3 Units.
The theory of linear and nonlinear partial differential equations, beginning with linear theory involving use of Fourier transform and Sobolev spaces. Topics: Schauder and L2 estimates for elliptic and parabolic equations; De Giorgi-Nash-Moser theory for elliptic equations; nonlinear equations such as the minimal surface equation, geometric flow problems, and nonlinear hyperbolic equations.

MATH 256B. Partial Differential Equations. 3 Units.
Continuation of 256A.

MATH 257A. Symplectic Geometry and Topology. 3 Units.
Linear symplectic geometry and linear Hamiltonian systems. Symplectic manifolds and their Lagrangian submanifolds, local properties. Symplectic geometry and mechanics. Contact geometry and contact manifolds. Relations between symplectic and contact manifolds. Hamiltonian systems with symmetries. Momentum map and its properties. May be repeated for credit.

MATH 257B. Symplectic Geometry and Topology. 3 Units.
Continuation of 257A. May be repeated for credit.

MATH 257C. Symplectic Geometry and Topology. 3 Units.
Continuation of 257B. May be repeated for credit.

MATH 258. Topics in Geometric Analysis. 3 Units.
May be repeated for credit.

MATH 262. Applied Fourier Analysis and Elements of Modern Signal Processing. 3 Units.
Introduction to the mathematics of the Fourier transform and how it arises in a number of imaging problems. Mathematical topics include the Fourier transform, the Plancherel theorem, Fourier series, the Shannon sampling theorem, the discrete Fourier transform, and the spectral representation of stationary stochastic processes. Computational topics include fast Fourier transforms (FFT) and nonuniform FFTs. Applications include Fourier imaging (the theory of diffraction, computed tomography, and magnetic resonance imaging) and the theory of compressive sensing.

MATH 263A. Algebraic Combinatorics and Symmetric Functions. 3 Units.
Symmetric function theory unifies large parts of combinatorics. Theorems about permutations, partitions, and graphs now follow in a unified way. Topics: The usual bases (monomial, elementary, complete, and power sums). Schur functions. Representation theory of the symmetric group. Littlewood-Richardson rule, quasi-symmetric functions, combinatorial Hopf algebras, introduction to Macdonald polynomials. Throughout, emphasis is placed on applications (e.g. to card shuffling and random matrix theory). Prerequisite: 210A and 210B, or equivalent.

MATH 263B. Crystal Bases: Representations and Combinatorics. 3 Units.
Crystal Bases are combinatorial analogs of representation theory of Lie groups. We will explore different aspects of these analogies and develop rigorous purely combinatorial foundations.

MATH 263C. Topics in Representation Theory. 3 Units.
May be repeated for credit.

MATH 269. Topics in symplectic geometry. 3 Units.
May be repeated for credit.

MATH 270. Geometry and Topology of Complex Manifolds. 3 Units.
Complex manifolds, Kahler manifolds, curvature, Hodge theory, Lefschetz theorem, Kahler-Einstein equation, Hermitian-Einstein equations, deformation of complex structures. May be repeated for credit.

MATH 271. The H-Principle. 3 Units.

MATH 272. Topics in Partial Differential Equations. 3 Units.

MATH 273. Topics in Mathematical Physics. 3 Units.
Covers a list of topics in mathematical physics. The specific topics may vary from year to year, depending on the instructor's discretion. Background in graduate level probability theory and analysis is desirable. Same as: STATS 359

MATH 280. Evolution Equations in Differential Geometry. 3 Units.

MATH 282A. Low Dimensional Topology. 3 Units.
The theory of surfaces and 3-manifolds. Curves on surfaces, the classification of diffeomorphisms of surfaces, and Teichmuller space. The mapping class group and the braid group. Knot theory, including knot invariants. Decomposition of 3-manifolds: triangulations, Heegaard splittings, Dehn surgery. Loop theorem, sphere theorem, incompressible surfaces. Geometric structures, particularly hyperbolic structures on surfaces and 3-manifolds. May be repeated for credit up to 6 total units.

MATH 282B. Homotopy Theory. 3 Units.
Homotopy groups, fibrations, spectral sequences, simplicial methods, Dold-Thom theorem, models for loop spaces, homotopy limits and colimits, stable homotopy theory. May be repeated for credit up to 6 total units.

MATH 282C. Fiber Bundles and Cobordism. 3 Units.

MATH 283. Topics in Algebraic and Geometric Topology. 3 Units.
May be repeated for credit.

MATH 283A. Topics in Topology. 3 Units.

MATH 284. Topics in Geometric Topology. 3 Units.
Incompressible surfaces, irreducible manifolds, prime decomposition, Morse theory, Heegaard diagrams, Heegaard splittings, the Thurston norm, sutured manifold theory, Heegaard Floer homology, sutured Floer homology.

MATH 284A. Geometry and Topology in Dimension 3. 3 Units.
The Poincare conjecture and the uniformization of 3-manifolds. May be repeated for credit.

MATH 284B. Geometry and Topology in Dimension 3. 3 Units.
The Poincare conjecture and the uniformization of 3-manifolds. May be repeated for credit.

MATH 286. Topics in Differential Geometry. 3 Units.
May be repeated for credit.
MATH 298. Graduate Practical Training. 1 Unit.
Only for mathematics graduate students. Students obtain employment in a relevant industrial or research activity to enhance their professional experience. Students submit a concise report detailing work activities, problems worked on, and key results. May be repeated for credit up to 3 units. Prerequisite: qualified offer of employment and consent of department. Prior approval by Math Department is required; you must contact the Math Department's Student Services staff for instructions before being granted permission to enroll.

MATH 301. Advanced Topics in Convex Optimization. 3 Units.
Modern developments in convex optimization: semidefinite programming; novel and efficient first-order algorithms for smooth and nonsmooth convex optimization. Emphasis on numerical methods suitable for large scale problems arising in science and engineering. Prerequisites: convex optimization (EE 364), linear algebra (Math 104), numerical linear algebra (CME 302); background in probability, statistics, real analysis and numerical optimization.
Same as: CME 375

MATH 305. Applied mathematics through toys and magic. 3 Units.
This course is a series of case-studies in doing applied mathematics on surprising phenomena we notice in daily life. Almost every class will show demos of these phenomena (toys and magic) and suggest open projects. The topics range over a great variety and cut across areas traditionally pigeonholed as physics, biology, engineering, computer science, mathematics but, instead, of developing sophisticated mathematics on simple material, our aim is to extract simple mathematical understanding from sophisticated material which, at first, we may not yet know how to pigeonhole. In each class I will try to make the discussion self-contained and to give everybody something to take home, regardless of the background.

MATH 355. Graduate Teaching Seminar. 1 Unit.
Required of and limited to first-year Mathematics graduate students.

MATH 360. Advanced Reading and Research. 1-10 Unit.

MATH 382. Qualifying Examination Seminar. 1-3 Unit.

MATH 391. Research Seminar in Logic. 1-3 Unit.

MATH 398. Readings in Mathematics. 1-10 Unit.

MATH 400A. Senior Project. 1-10 Unit.
This course is devoted to independent research or extended projects. Students submit a concise report detailing work activities, problems worked on, and key results. May be repeated for credit up to 10 units. Prerequisite: qualified offer of employment and consent of the instructor.

MATH 400B. Senior Project. 1-10 Unit.

MATH 410A. Research Seminar. 3-4 Unit.

MATH 410B. Research Seminar. 3-4 Unit.

MATH 410C. Research Seminar. 3-4 Unit.

MATH 410D. Research Seminar. 3-4 Unit.

MATH 410E. Research Seminar. 3-4 Unit.

MATH 410F. Research Seminar. 3-4 Unit.

MATH 410G. Research Seminar. 3-4 Unit.

MATH 410H. Research Seminar. 3-4 Unit.

MATH 410I. Research Seminar. 3-4 Unit.

MATH 410J. Research Seminar. 3-4 Unit.

MATH 410K. Research Seminar. 3-4 Unit.

MATH 410L. Research Seminar. 3-4 Unit.

MATH 410M. Research Seminar. 3-4 Unit.

MATH 410N. Research Seminar. 3-4 Unit.

MATH 410O. Research Seminar. 3-4 Unit.

MATH 410P. Research Seminar. 3-4 Unit.

MATH 410Q. Research Seminar. 3-4 Unit.

MATH 410R. Research Seminar. 3-4 Unit.

MATH 410S. Research Seminar. 3-4 Unit.

MATH 410T. Research Seminar. 3-4 Unit.

MATH 410U. Research Seminar. 3-4 Unit.

MATH 410V. Research Seminar. 3-4 Unit.

MATH 410W. Research Seminar. 3-4 Unit.

MATH 410X. Research Seminar. 3-4 Unit.

MATH 410Y. Research Seminar. 3-4 Unit.

MATH 410Z. Research Seminar. 3-4 Unit.

MATH 411A. Graduate Students Seminar: Theory of Probability. 1-3 Unit.

MATH 425A. Group Theory. 3 Units.
Elementary theory of groups. Subgroups, cosets, homomorphisms, normal subgroups and quotients, classes of groups (abelian, nilpotent, solvable), simple groups, Sylow theorems, free groups. Group actions. Prerequisite: linear algebra and group theory. Prerequisite: Math 61CM.

MATH 425B. Group Theory. 3 Units.

different themes. Prerequisite: Math 61CM and Math 62CM. Topics include linear systems of differential equations and necessary tools from linear algebra, stability and asymptotic properties of solutions to linear systems, existence and uniqueness theorems for nonlinear differential equations with some applications to manifolds, behavior of solutions near an equilibrium point, and Sturm-Liouville theory. Prerequisites: Math 61CM and Math 62CM.

MATH 425C. Group Theory. 3 Units.

MATH 51A. Linear Algebra and Differential Calculus of Several Variables. 6 Units.
Students attend MATH 51 lectures with different recitation sessions: three hours per week instead of two, emphasizing engineering applications. Prerequisite: application; see https://web.stanford.edu/dept/soe/osa/ace.fb.

MATH 52. Integral Calculus of Several Variables. 5 Units.
Iterated integrals, line and surface integrals, vector analysis with applications to vector potentials and conservative vector fields, physical interpretations. Divergence theorem and the theorems of Green, Gauss, and Stokes. Prerequisite: 51 or equivalents.

MATH 53. Ordinary Differential Equations with Linear Algebra. 5 Units.
Ordinary differential equations and initial value problems, systems of linear differential equations with constant coefficients, applications of second-order equations to oscillations, matrix exponentials, Laplace transforms, stability of non-linear systems and phase plane analysis, numerical methods. Prerequisite: 51 or equivalents.

MATH 61CM. Modern Mathematics: Continuous Methods. 5 Units.
This is the first part of a theoretical (i.e., proof-based) sequence in multivariable calculus and linear algebra, providing a unified treatment of these topics. Covers general vector spaces, linear maps and duality, eigenvalues, inner product spaces, spectral theorem, metric spaces, differentiation in Euclidean space, submanifolds of Euclidean space, inverse and implicit function theorems, and many examples. The linear algebra content is covered jointly with Math 61DM. Students should know 1-variable calculus and have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on the BC-level Advanced Placement calculus exam, or consent of the instructor.

MATH 61DM. Modern Mathematics: Discrete Methods. 5 Units.
This is the first part of a theoretical (i.e., proof-based) sequence in discrete mathematics and linear algebra. Covers general vector spaces, linear maps and duality, eigenvalues, inner product spaces, spectral theorem, counting techniques, and linear algebra methods in discrete mathematics including spectral graph theory and dimension arguments. The linear algebra content is covered jointly with Math 61CM. Students should have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on the BC-level Advanced Placement calculus exam, or consent of the instructor.

MATH 62CM. Modern Mathematics: Continuous Methods. 5 Units.
A continuation of themes from Math 61CM, centered around: manifolds, multivariable integration, and the general Stokes' theorem. This includes a treatment of multilinear algebra, further study of submanifolds of Euclidean space and an introduction to general manifolds (with many examples), differential forms and their geometric interpretations, integration of differential forms, Stokes' theorem, and some applications to topology. Prerequisite: Math 61CM.

MATH 62DM. Modern Mathematics: Discrete Methods. 5 Units.
This is the second part of a proof-based sequence in discrete mathematics. This course covers topics in elementary number theory, group theory, and discrete Fourier analysis. For example, we'll discuss the basic examples of abelian groups arising from congruences in elementary number theory as well as the non-abelian symmetric group of permutations. Prerequisites: 61DM or 61CM.

MATH 63CM. Modern Mathematics: Continuous Methods. 5 Units.
A proof-based course on ordinary differential equations, continuing themes from Math 61CM and Math 62CM. Topics include linear systems of differential equations and necessary tools from linear algebra, stability and asymptotic properties of solutions to linear systems, existence and uniqueness theorems for nonlinear differential equations with some applications to manifolds, behavior of solutions near an equilibrium point, and Sturm-Liouville theory. Prerequisites: Math 61CM and Math 62CM.

MATH 63DM. Modern Mathematics: Discrete Methods. 5 Units.
Third part of a proof-based sequence in discrete mathematics. This course covers several topics in probability (random variables, independence and correlation, concentration bounds, the central limit theorem) and topology (metric spaces, point-set topology, continuous maps, compactness, Brouwer's fixed point and the Borsuk-Ulam theorem), with some applications in combinatorics. Prerequisites: 61DM or 61CM.

MATH 70SI. The Game of Go: Strategy, Theory, and History. 1 Unit.
Strategy and mathematical theories of the game of Go, with guest appearance by a professional Go player.
MATH 79SI. Proof Positive: Principles of Mathematics. 1 Unit.
What is a mathematical proof, and where do proofs come from? Students will become comfortable with fundamental techniques of mathematical proof through practice with interesting and accessible examples from many areas of math. Students will additionally hone their communication skills and develop their ability to formulate and answer precise mathematical questions. Topics include direct proof, proof by contrapositive, proof by contradiction, many applications of mathematical induction, constructing good definitions, and useful writing habits. The course is designed to prepare students who have completed or are concurrently enrolled in MATH 51 to succeed in introductory proof-based math classes at the level of MATH 115 or MATH 120, or to simply appreciate the nature of proof at a deeper level than is seen in high school geometry. To be considered for enrollment, please email masonr@stanford.edu and attend the first class meeting on Tuesday, April 3 at 3PM in 300-303.

MATH 802. TGR Dissertation. 0 Units.

MATH 80Q. Capillary Surfaces: Explored and Unexplored Territory. 3 Units.
Preference to sophomores. Capillary surfaces: the interfaces between fluids that are adjacent to each other and do not mix. Recently discovered phenomena, predicted mathematically and subsequently confirmed by experiments, some done in space shuttles. Interested students may participate in ongoing investigations with affinity between mathematics and physics.

MATH 83N. PROOFS AND MODERN MATHEMATICS. 3 Units.
How do mathematicians think? Why are the mathematical facts learned in school true? In this course students will explore higher-level mathematical thinking and will gain familiarity with a crucial aspect of mathematics: achieving certainty via mathematical proofs, a creative activity of figuring out what should be true and why. This course is ideal for students who would like to learn about the reasoning underlying mathematical results, but at a pace and level of abstraction not as intense as Math 61CM/DM, as a consequence benefiting from additional opportunity to explore the reasoning. Familiarity with one-variable calculus is strongly recommended at least at the AB level of AP Calculus since a significant part of the seminar develops some of the main results in that material systematically from a small list of axioms. We also address linear algebra from the viewpoint of a mathematician, illuminating algebraic notions such as groups, rings, and fields. This seminar may be paired with Math 51; though that course is not a pre- or co-requisite.

MATH 87Q. Mathematics of Knots, Braids, Links, and Tangles. 3 Units.
Preference to sophomores. Types of knots and how knots can be distinguished from one another by means of numerical or polynomial invariants. The geometry and algebra of braids, including their relationships to knots. Topology of surfaces. Brief summary of applications to biology, chemistry, and physics.